HIGH SCHOOL OF ENGINEERING

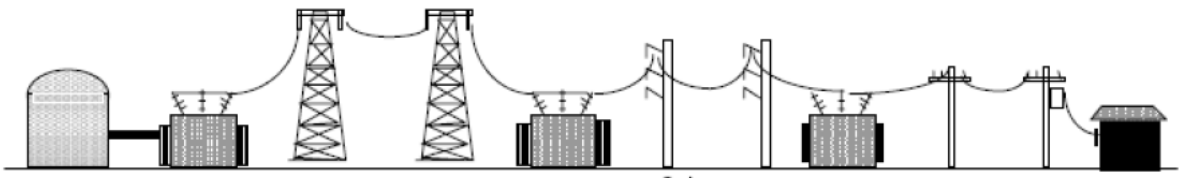
THE GENERAL CYCLE– HEI 3

Electrotechnical courses

Part 1

REFERENCE: 3A41-1

Electrical Energy Systems



HEI 2015/2016
13, rue de Toul
59 046 Lille Cedex

M. Nasser

Phone. : 03 28 38 48 58
Fax : 03 28 38 48 04
Web: www.hei.fr
E-mail : ecole@hei.fr

Table of contents

INTRODUCTION

This course is designed to give you an overview of the electrical and electronics technology that is currently used in the industry. It is not intended to go into sufficient depth and detail for you to be able to design electric motors, or electronic monitoring systems or computer controllers, because most of you will not even come close to doing such things.

However, many of you will, as managers, have to deal with the people who do design, run and maintain electronic equipment. You need to understand what they are talking about so you can discuss problems confidently and understand the technology sufficiently well to specify new equipment and buy appropriate machinery when the time comes.

Quite a few of you will probably end up doing technical management, technical development or research, so the lecture course is very important, because it will give you:

- Basic knowledge about electrical circuit theory, circuit analysis methods and operating principles of electric machines
- Students are allowed a lot of latitude in choosing elective courses, and this gives them the possibility to become either « non-specialized » or else « specialists » in a given field
- It opens up a variety of job perspectives

CHAPTER ONE

Complex Numbers

Learning Outcomes

As a result of studying this topic, students will be able to

• add and subtract Complex Numbers and to appreciate that the addition of a Complex Number to another Complex Number corresponds to a translation in the plane

• multiply Complex Numbers and show that multiplication of a Complex Number by another Complex Number corresponds to a rotation and a scaling of the Complex Number

• find the conjugate of a Complex Number

• divide two Complex Numbers and understand the connection between division and multiplication of Complex Numbers

While the fundamental signal used in electrical engineering is the **sinusoid**, it can be expressed mathematically in terms of an even more fundamental signal: the **complex exponential**

Introduction

The study of complex numbers began to find roots of the polynomial equation $x^2+1 = 0$.

It turns out that this equation does not have any real root. Thus one needs to go out of real numbers to get a solution of the above equation.

To do all these, it turns out that one needs also to study functions defined on the set of complex numbers. That is, we need to do calculus of functions (differential calculus and integral calculus) defined on the set of complex numbers.

Complex numbers

While the fundamental signal used in electrical engineering is the **sinusoid**, it can be expressed mathematically in terms of an even more fundamental signal: the **complex exponential**

A complex number, $\underline{z}$ consists of the ordered pair (a, b), **a** is the **real** component and **b** is the **imaginary** component (the **j** is suppressed because the imaginary component of the pair is always in the second position). The imaginary number **jb** equals (0, b). Note that **a** and **b** are real-valued numbers.

Figure 1 (The Complex Plane) shows that we can locate a complex number in what we call the complex plane. Here, **a**, the real part, is the $\Re$ **-coordinate** and **b**, the imaginary part, is the $\Im$ **-coordinate**. Consequently, a complex number $\underline{z}$ can be expressed as the (vector) sum $\underline{z} = a + jb$ where j indicates the $\Im$-coordinate. This representation is known as **the Cartesian form of** $\underline{z}$. An imaginary number can't be numerically added to a real number; rather, this notation for a complex number represents vector addition, but it provides a convenient notation when we perform arithmetic manipulations. Figure 1-shows a complex Plane of $\underline{z}$.

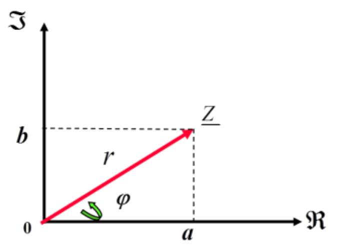


Figure 1 the Complex Plane of $\underline{z}$

The complex conjugate of $\underline{z}$, written as $\underline{z}^*$, has the same real part as $\underline{z}$ but an imaginary part of the opposite sign.

$$\underline{z} = \Re[\underline{z}] + j\Im[\underline{z}]$$

$$\underline{Z}^* = \Re[\underline{Z}] - j\Im[\underline{Z}]$$

By forming a right triangle having sides **a** and **b**, we see that the real and imaginary parts correspond to the cosine and sine of the triangle's base angle. We thus obtain **the polar form** for complex numbers.

$$\underline{Z} = a + jb = r \times e^{j\varphi} \quad r = |\underline{Z}| = \sqrt{a^2 + b^2} \quad a = |\underline{Z}|\cos\varphi \quad b = |\underline{Z}|\sin\varphi \quad \varphi = arctag(\frac{b}{a})$$

The quantity **r** is known as the **magnitude** of the complex number $\underline{Z}$, and is frequently written as $|\underline{Z}|$. The quantity φ is the **complex number's angle**. In using the arc-tangent formula to find the angle, we must take into account the quadrant in which the complex number lies.

Exercise 1
Convert $\underline{Z} = 3 + 2j$ to **polar** form.

Calculating with Complex Numbers

- **Addition and subtraction**

Using Cartesian notation, the following properties easily follow.

- If we **add** two complex numbers, the real part of the result equals the **sum** of the real parts and the imaginary part equals the **sum** of the imaginary parts. This property follows from the laws of vector addition.

  $\underline{Z} = \underline{Z}_1 + \underline{Z}_2 = (a + jb) + (c + jd) = (a + c) + j(b + d)$

  In this way, the real and imaginary parts remain separate.

Exercise 2.
What is the sum of a complex number $\underline{Z}$? Given $\underline{Z}_1 = 3 - 2j$; $\underline{Z}_2 = 5 + 4j$. Required $\underline{Z} = \underline{Z}_1 + \underline{Z}_2$

- **Multiplication**

To **multiply**, the **radius** equals the **product** of the **radii** and the **angle** the **sum** of the angles.

$\underline{Z} = \underline{Z}_1 \times \underline{Z}_2$

$\underline{Z} = (a + jb) \times (c + jd) = \sqrt{a^2 + b^2} \times \sqrt{c^2 + d^2} \left[(\cos(\varphi_1 + \varphi_2) + j\sin(\varphi_1 + \varphi_2)\right])$

Exercise 3.
What is the product of a complex number $\underline{Z}$? Given $\underline{Z}_1 = \sqrt{3} - j; \underline{Z}_2 = 1 - j$. Required $\underline{Z} = \underline{Z}_1 \times \underline{Z}_2$

- **Division**

To **divide**, the **radius** equals the **ratio** of the **radii** and the **angle** the **difference** of the angles

$$\underline{Z} = \frac{\underline{Z}_1}{\underline{Z}_2} = \frac{a + jb}{c + jd}$$

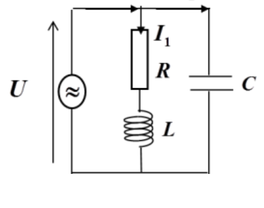
$$|\underline{Z}| = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}$$

$$\varphi = arctg(\frac{b}{a}) - arctg(\frac{d}{c})$$

Exercise 4.
What is the ratio of a complex number $\underline{Z}$? Given $\underline{Z}_1 = \sqrt{3} - j; \underline{Z}_2 = 1 - j$. Required $\underline{Z} = \frac{\underline{Z}_1}{\underline{Z}_2}$

Exercise 5.
A circuit is set up as shown below. The frequency of the source is 50Hz

$U = 48V$ $f = 50Hz$
$R = 50\Omega$ $L = 200mH$ $C = 10\mu F$



Calculate for the circuit:

a) The impedance $\underline{Z_1} = (R + jL\omega)$;

b) The impedance $\underline{Z_2} = \dfrac{1}{jC\omega}$

c) The RMS currents and complex number's angle $(\underline{I_1}; \underline{I_2}; \underline{I})$

Observation:
An imaginary number can't be numerically added to a real number rather, this notation for a complex number represents vector addition.

Exercise 6

A dipole is designed for use with a $v(t) = 4\sqrt{2}\sin(314t + 0{,}524)$ and
$i(t) = 0{,}127\sqrt{2}\cos(314t - 1{,}047)$

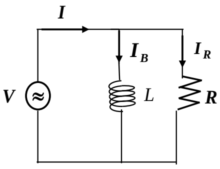
Calculate for the circuit:

a) The impedance $\underline{Z}$;

b) The inductance L

Exercise 7

A circuit is set up as shown below. The frequency of the source is 50Hz. Given $I_B = 2A; I_R = 1A$

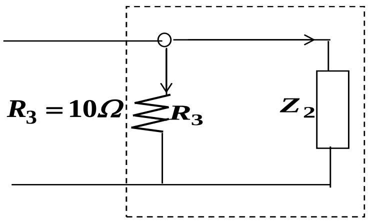


Calculate for the circuit:

a) The RMS current and phase shift of $(\underline{I})$

Exercise 8

A circuit is set up as shown below. The frequency of the source is 50Hz and



$Z_2 = (10 + j15)\Omega$

Calculate for the circuit:

a) The impedance; $Z = R_3 // Z_2$

Keywords

Real component: Partie réelle
Imaginary component: Partie imaginaire
Complex number: Nombres complexes
Complex Plane: Plan complexe
The Cartesian form: Représentation algébrique
The polar form: Représentation exponentielle
The complex conjugate: Nombres complexes conjugués
An imaginary number can't be numerically added to a real number

Basic Formulas

$$\underline{Z} = a + jb = r(\cos\varphi + j\sin\varphi) = r \times e^{j\varphi}$$

$$r = |\underline{Z}| = \sqrt{a^2 + b^2}$$

$$\varphi = arctag(\frac{b}{a})$$

- **Addition and subtraction**

$$\underline{Z} = \underline{Z_1} + \underline{Z_2} = (a + jb) + (c + jd) = (a + c) + j(b + d)$$

- **Multiplication**

$$\underline{Z} = (a + jb) \times (c + jd) = \sqrt{a^2 + b^2} \times \sqrt{c^2 + d^2}\left[(\cos(\varphi_1 + \varphi_2) + j\sin(\varphi_1 + \varphi_2)\right])$$

$$\varphi = \varphi_1 + \varphi_2$$

- **Division**

$$|\underline{Z}| = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}$$

$$\varphi = arctg(\frac{b}{a}) - arctg(\frac{d}{c})$$

CHAPTER TWO

Alternative current (AC) Sine Wave

Single-phase AC power

Learning Outcomes:
After successfully studying this course, students will be able to:
1. Systematically obtain the equations that characterize the performance of an electric circuit as solving single phase circuits in sinusoidal steady state.
2. Acquire skills in using electrical measuring devices.
3. Simple AC series and parallel circuits are analyzed using phasor and fundamental circuit laws.
4. Perform calculations with power in AC circuits. Power and power factor for simple AC circuits are calculated

Introduction

WHY AC? Importance of AC

- Electric power is generated, transmitted, distributed and consumed in AC
- ≈ 90% of the total electric power is consumed as AC

Single-phase power is used in homes, offices, and many other types of facilities.
MEASUREMENTS OF AC MAGNITUDES
The voltage waveform produced as the armature of a basic two-pole AC generator rotates through 360 degrees is called a sine wave because the instantaneous voltage or current is related to the sine trigonometric function
Alternating voltage and current vary continuously. The graphic representation for AC is a sine wave. A sine wave can represent current or voltage. There are two axes. The vertical axis represents the direction and magnitude of current or voltage. The horizontal axis represents time. Figure 1- shows a sine wave

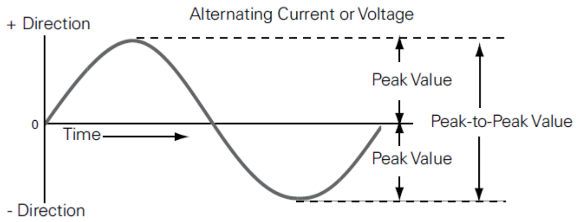


Figure 1. The graphical representation of a sine wave

Voltage
The force required to make electricity flow through a conductor is called a difference in potential, electromotive force (emf), or voltage. Voltage is designated by the letter V.
The unit of measurement for voltage is the volt.

Frequency
The number of cycles per second of voltage or current
The recognized unit for frequency is hertz; abbreviated Hz. 1 Hz is equal to 1 cycle per second

Sine definition

$$\underline{v}(t) = Ae^{j\left(\omega t+\delta+\frac{\pi}{2}\right)} = A(\sin(\omega t + \delta) + j\cos(\omega t + \delta))$$

$$v(t) = \Re(\underline{v}(t)) = A\sin(\omega t + \delta)$$

What is the amplitude, pulse wave, frequency, period and angle at the origin of $v(t) = 325\sin(314t + 1)$

Amplitude
The amplitude is the range of variation. Amplitude can be specified in three ways: peak value, peak-to peak value, and effective value
The **peak** value of a sine wave is **the maximum** value for each half of the sine wave
The **peak-to-peak** value is the range from the positive peak to the negative peak. This is twice the peak value.

Average value = 0 $< v(t) >= \frac{1}{T}\int_0^T v(t).dt = 0$

The <u>**effective**</u> value of AC is defined in terms of an equivalent heating effect when compared to DC

Root Mean Square Value (RMS)

$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = \frac{V_{peak}}{\sqrt{2}}$. The effective value is also referred to as the RMS (root-mean-square value)

Instantaneous Value

The instantaneous value is the value at any one point on the sine wave
As shown in the following illustration, the instantaneous voltage $v(t)$ and current $i(t)$ at any point on the sine wave are equal to the peak value times the sine of the angle. The sine values shown in the illustration are obtained from trigonometric tables. Keep in mind that each point has an instantaneous value, but this illustration only shows the sine of the angle at 30 degree intervals. The sine of an angle is represented symbolically as $\sin\theta$, where the Greek letter theta (θ) represents the angle Complex Plane of $\underline{Z}$. Figure 2- shows an instantaneous value of alternating current or voltage.

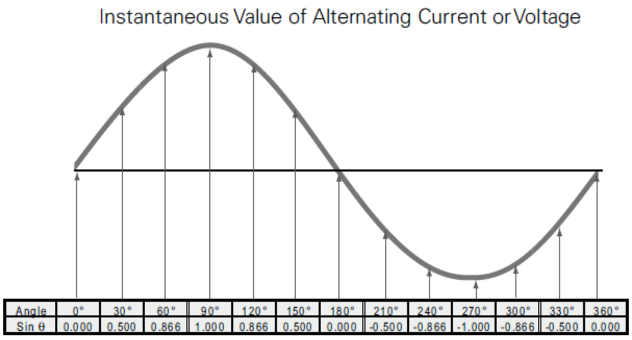


Figure 2- An instantaneous value of alternating current or voltage.

Instantaneous current $i(t) = I_{peak}\sin\theta$

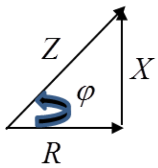
Instantaneous voltage $v(t) = V_{peak}\sin\theta$

Example: if Vpeak = 325, 26 V, at 150 degrees $v(t) = 0{,}5 \times V_{peak} = 162{,}63V$

<u>**Impedance**</u>

Total opposition to current flow in an AC circuit that contains both reactance and resistance is called <u>impedance</u>, designated by the symbol Z. Just like resistance, reactance and impedance are expressed in ohms. $\underline{Z} = R + jX$

Impedance triangle



$|\underline{Z}| = \sqrt{R^2 + X^2}$ $R = |\underline{Z}|\cos\varphi$ $X = |\underline{Z}|\sin\varphi$

PHASE IN AC CIRCUITS

Out-of-phase waveforms (Figure 3)

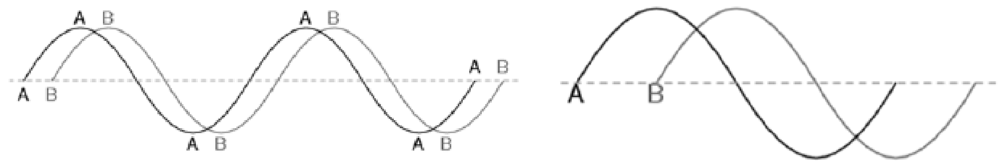


Figure 3. **Out-of-phase** waveforms
Phase shift of 90 degrees:
A leads B
B lags A

AC PURE RESISTIVE CIRCUITS

Voltage and current are "***in phase***" (Figure 4)

- Instantaneous AC power is always positive.
- $\varphi = 0$ Phase shift between voltage and current $\cos\varphi = 1$

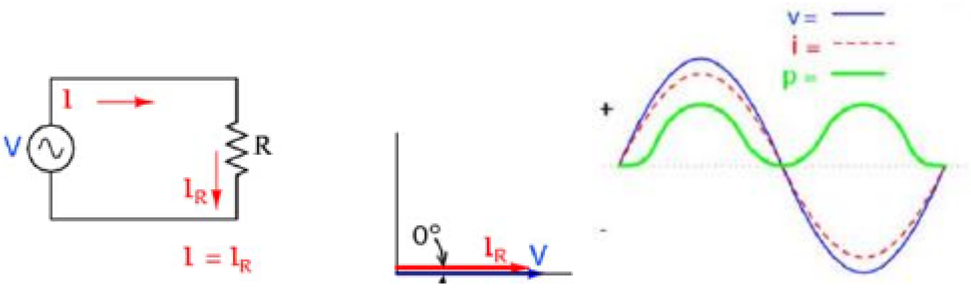


Figure 4. Voltage and current are "***in phase***"

$v(t) = V_{peak} \sin \omega t$

A phase vector representation is: $V = RI$

AC PURE INDUCTIVE CIRCUITS

Inductor **current <u>lags</u> inductor voltage** by 90º (Figure 5)
The current reaches its maximum value up to 90° **<u>behind</u>** the voltage
Instantaneous AC power is positive or negative

$\varphi = 90^{\circ}$ Phase shift between voltage and current $\cos\varphi = 0$

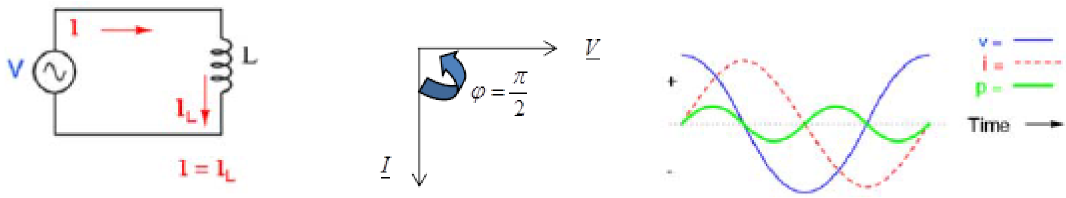


Figure 5. Inductor current lags inductor voltage by 90º

$v(t) = L\dfrac{di}{dt}$ A phase vector representation is: $\underline{V} = jL\omega\underline{I}$

AC PURE CAPACITIVE CIRCUITS

Capacitor **current <u>leads</u> capacitor voltage by** 90(Figure 6)
The current reaches its maximum value up to 90° **<u>ahead</u>** of the voltage
Instantaneous AC power may be positive or negative

$\varphi = -90^{\circ}$ Phase shift between voltage and current $\cos\varphi = 0$

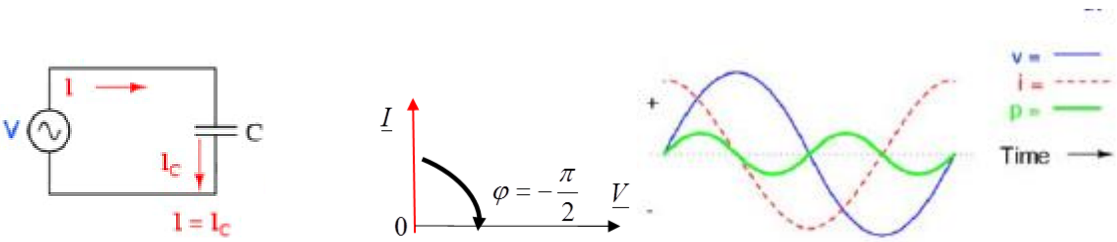


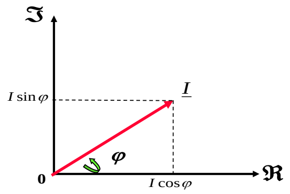
Figure 6 Capacitor voltage lags capacitor current by 90º

$$i(t) = C\frac{dv}{dt}$$

A phase vector representation is: $\underline{V} = -j\frac{1}{C\omega}\underline{I}$

Active and Reactive Current Component

Consider a branch in a network with $\underline{V}$ the phasor of the branch voltage and $\underline{I}$ the current phasor.



The **active current** I_a and **reactive current** I_r are defined as the current components in phase and in quadrature with the branch voltage.
They are explicitly given by:

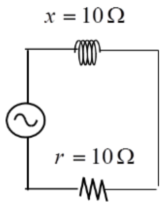
$I_a = I\cos\varphi \qquad I_r = I\sin\varphi \qquad \underline{I} = I\cos\varphi + j\sin\varphi$

Review 1

1. A __________ is the graphic representation of AC voltage or current values over time.
2. An AC generator produces ____________ cycle(s) per revolution for each pair of poles.
3. The instantaneous voltage at 240 degrees for a sine wave with a peak voltage of 150 V is ________ V.
4. The effective voltage for a sine wave with a peak voltage of 150 V is ________ V.

Examples:

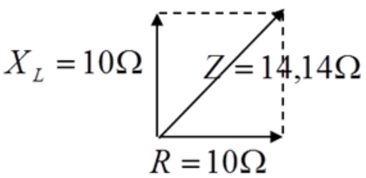
1. For example, if resistance and inductive reactance are each 10Ω, impedance is calculated as follows



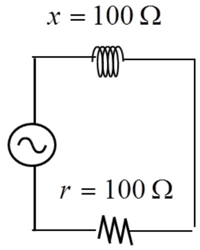
Inductor **current <u>lags</u> inductor voltage**

The following formula is used to calculate impedance in a circuit with resistance and inductive reactance. $|\underline{Z}| = \sqrt{R^2 + X_L^2}$

$|\underline{Z}| = \sqrt{10^2 + 10^2} = 14{,}14\Omega$. A common way to represent AC circuit values is with a vector diagram.

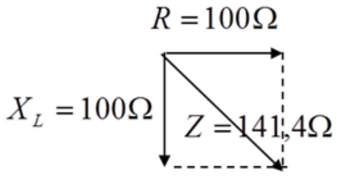


1. For example, if resistance and capacitive reactance are each 100Ω, impedance is calculated as follows



Capacitor **current <u>leads</u> capacitor voltage**
The following formula is used to calculate impedance in a circuit with resistance and capacitive reactance. $|\underline{Z}| = \sqrt{R^2 + X_c^2}$

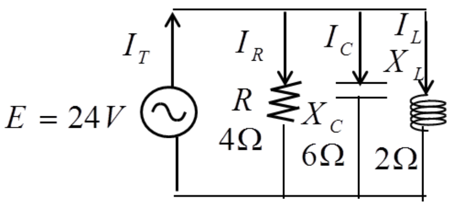
$|\underline{Z}| = \sqrt{10^2 + 10^2} = 141{,}4\Omega$. The following vector illustrates the relationship between resistance and capacitive reactance for a circuit containing 100Ω of each.



Reactance: In an AC circuit with only inductance, capacitance, or both inductance and capacitance, but no resistance, opposition to current flow is called reactance, designated by the symbol X.
$X = \mathrm{Im}(\underline{Z}) = Z \sin\varphi$

<u>Exercise1</u>
A circuit is set up as shown below. The frequency of the source is 50Hz
Calculate for the circuit:

a) The current $\underline{I_R}$;
b) The current $\underline{I_C}$;
c) The current $\underline{I_L}$;
d) The current $\underline{I_T}$;
e) The impedance $\underline{Z_T}$;



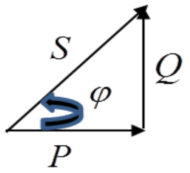
Power and Power Factor in an AC Circuit

Power consumed by a resistor is dissipated in heat and not returned to the source. This is called **<u>true power</u>** because it is the rate at which energy is used.
Current in an AC circuit rises to peak values and diminishes to zero many times a second. The energy stored in the magnetic field of an inductor, or plates of a capacitor, is returned to the source when current changes direction. The formula for true power is shown below. The unit of measure for true power is the Watt (W). $P = VI\cos\varphi$

Although reactive components do not consume energy, they do increase the amount of energy that must be generated to do the same amount of work. The rate at which this non-working energy must be generated is called **<u>reactive power</u>**. If voltage and current are 90 degrees out of phase, as would be the case in a purely capacitive or purely inductive circuit, the average value of true power is equal to zero. In this case, there are high positive and negative peak values of power, but when added together the result is zero. The formula for reactive power is shown below. The unit of measure for reactive power is the volt-ampere-reactive (VAR). $Q = VI\sin\varphi$

Power in an AC circuit is the vector sum of true power and reactive power. This is called apparent power. True power is equal to apparent power in a purely resistive circuit because voltage and current are in phase. Voltage and current are also in phase in a circuit containing equal values of inductive reactance and capacitive reactance. In most circuits, however, **apparent power** is composed of both true power and reactive power. The formula for apparent power is shown below. The unit of measure for apparent power is the volt-ampere (VA). $\underline{S} = P + jQ$ $\quad \underline{S} = \underline{V} \times \underline{I}^*$

Power triangle



$|\underline{S}| = \sqrt{P^2 + Q^2}$ $\quad P = |\underline{S}|\cos\varphi$ $\quad Q = |\underline{S}|\sin\varphi$

In a purely resistive circuit, current and voltage are in phase and there is a zero degree angle displacement between current and voltage. Therefore, in a purely resistive circuit, there is not a **reactive power**.

In a purely reactive circuit, either inductive or capacitive, current or voltages are 90 degrees out of phase. The cosine of 90 degrees is zero. Therefore, in a purely reactive circuit there is not a **true power**. Therefore, no energy is consumed in a purely reactive circuit.

Complex impedance method for AC circuits (Table 1)

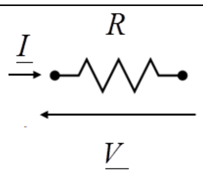
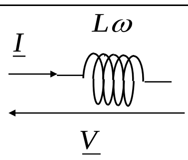
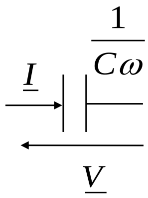
	RESISTOR	INDUCTOR	CAPACITOR
			
Impedance (Ω)	$Z = R$	$Z = L\omega$	$Z = \frac{1}{C.\omega}$
Phase shift (rad)	$\varphi = 0$	$\varphi = +\frac{\pi}{2}$	$\varphi = -\frac{\pi}{2}$
Complex impedance (Ω)	$\underline{Z} = R$	$\underline{Z} = jL\omega$	$\underline{Z} = -j\frac{1}{C.\omega}$
Active Power (W)	$P = R.I^2$	$P = 0$	$P = 0$
Reactive Power (VAR)	$Q = 0$	$Q = L\omega.I^2$	$Q = -C\omega.V^2$
Apparent Power (VA)	$\underline{S} = P$	$\underline{S} = jQ$	$\underline{S} = jQ$

Table 1. Complex impedance

Power Calculation Example

Power Factor

Power factor is the ratio of true power to apparent power in an AC circuit. As previously indicated, this ratio is also the cosine of the phase angle. $\cos\varphi = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}$

In a purely resistive circuit, current and voltage are in phase. This means that there is no angle of displacement between current and voltage. The cosine of a zero degree angle is one. Therefore, the power factor is one. This means that all energy delivered by the source is consumed by the circuit and dissipated in the form of heat.

In a purely reactive circuit, voltage and current are 90 degrees apart. The cosine of a 90 degree angle is zero. Therefore, the power factor is zero. This means that all the energy the circuit receives from the source is returned to the source.
For the circuit in the following example, the power factor is 0.8. This means the circuit uses 80 percent of the energy supplied by the source and returns 20 percent to the source.

Compensation with capacities

To increase the power factor of the load, one can connect capacities in parallel to the load impedances. The reactive power generated by the capacity compensates a part of the inductive reactive power consumed by the load.

$$Q_c = -C\omega V^2 = Q' - Q$$

$$-C\omega V^2 = P\tan\varphi' - P\tan\varphi$$

Q : The uncompensated reactive power

Q' : The compensated reactive power

Q_c The reactive power generated by the capacity

P : The active power
We can determine the value of the capacity C. The capacity becomes

$$C = \frac{P(\tan\varphi - \tan\varphi')}{\omega V^2}$$

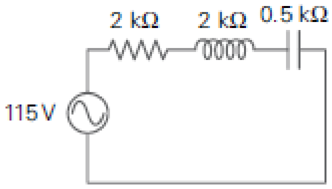
Effects of the compensation

- The phase current is reduced
- Line losses are reduced
- The amplitude and phase angle of the load voltage increase

Exercise 2

A circuit is set up as shown below. The frequency of the source is 50Hz
Calculate for the circuit:

a) The current $\underline{I}$;
b) The true power P ;
c) The apparent power S;
d) The power factor $\cos\varphi$;



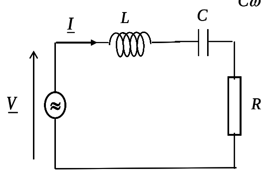
Review

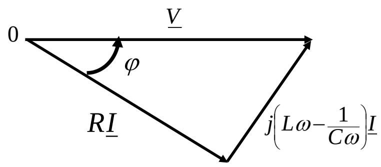
1. An AC circuit is ________ if inductive reactance and capacitive reactance are equal.
2. A series AC circuit is ________ if there is more inductive reactance than capacitive reactance.
3. A series AC circuit is ________ if there is more capacitive reactance than inductive reactance.
4. For a circuit with a 120 V AC source and a current of 10 A, the apparent power is __________ VA.
6. For a circuit with an apparent power of 3000 VA and a power factor of 0.8, the true power is ________ W.

Exercise 3

A 100 V AC power supply, 20Ω - resistors, 10Ω coil and -5Ω capacitor are in series.

$\underline{V} = 100e^{j0}V$ $f = 50Hz$ $R = 20\Omega$ $-\frac{1}{C\omega} = -5\Omega$ $L\omega = 10\Omega$





Calculate:

a) The RMS current of $(\underline{I})$ and phase shift (φ)

Voltage Drop Calculations

Because all conductors exhibit impedance to the flow of electric current, the voltage will not be constant throughout the system, but will tend to drop as one move closer to the load. Ohm's Law, expressed in phasor form for AC circuits, and gives the basic relationship for voltage drop and the load current.

$$\underline{\Delta V} = \underline{Z}_{cond} \times \underline{I}$$

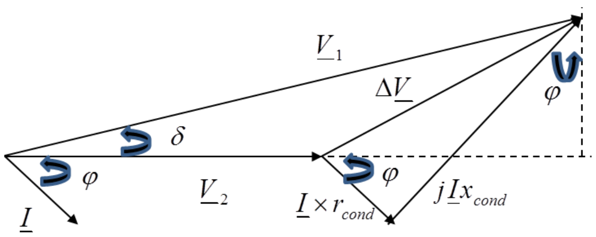
$$\underline{Z}_{cond} = (r_{cond} + jx_{cond})$$

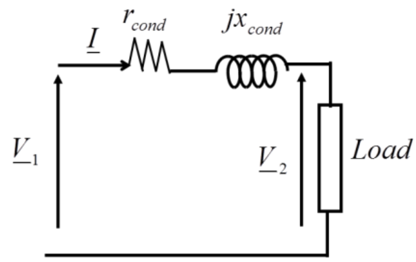
$\underline{\Delta V}$ is the voltage drop along a length of conductor or across a piece of equipment in **volts**
$\underline{I}$ is the load current in **amperes**

$\underline{Z}_{cond}$ is the conductor or equipment impedance, in **ohms**

Exercise 4. A circuit is set up as shown below. The frequency of the source is 50Hz

- Draw the Phasor Diagram of the circuit
- Calculate for the circuit the drop voltage $\underline{\Delta V} = \underline{V}_1 - \underline{V}_2$





- **Boucherot theorem or conservation theorem for complex power**

Power balance for circuit solution

The active power

$$P = \sum_{i=1}^{n} P_i;$$

The reactive power

$$Q = \sum_{i=1}^{n} Q_i$$

The global complex power of the loads becomes then:

$$\underline{S} = \sum_{i=1}^{n} \underline{S}_i$$

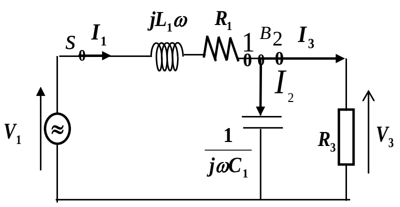
$$\underline{S} = P \pm jQ = \sum_{i=1}^{n} P_i \pm j \sum_{i=1}^{n} Q_i$$

- In this way, the real and imaginary parts remain separate.

Exercise 4

A circuit is set up as shown below. The frequency of the source is 50Hz. Given $V_3 = 228{,}31V$. Calculate for the circuit:

a) The voltage V_1;



$$V_3 = 228{,}31V; R_1 = 1\Omega; L_1\omega = 2\Omega; \frac{1}{C_1\omega} = 400\Omega; R_3 = 20\Omega$$

Review

1. ________ is the opposition to current flow in an AC circuit caused by inductance and capacitance.
2. ________ is the total opposition to current flow in an AC circuit with resistance, capacitance, and/or inductance.

3. For a 50 Hz circuit with a 10 mH inductor, the inductive reactance is _______ Ω.
4. In a purely inductive circuit, ________.
a. current and voltage are in phase
b. current leads voltage by 90 degrees
c. current lags voltage by 90 degrees
5. In a purely capacitive circuit, ________.
a. current and voltage are in phase
b. current leads voltage by 90 degrees
c. current lags voltage by 90 degrees
6. For a 50 Hz circuit with a $10\mu F$ capacitor, the capacitive reactance is ________ Ω.
7. A circuit with 5Ω of resistance and 10Ω of inductive reactance has an impedance of ________ Ω.
8. A circuit with 5Ω of resistance and 4Ω of capacitive reactance has an impedance of ________ Ω.

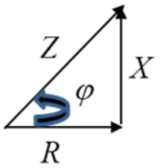
Keywords
Alternative current: Courant alternatif
Voltage: Tension
Frequency: Fréquence
Amplitude: amplitude
Instantaneous Value: valeur instantanée
Average value: Valeur efficace
Root Mean Square Value (RMS): Valeur efficace vraie
Impedance: impédance
Phase shift: angle de phase
In phase: En phase
Current lags voltage: Courant en arrière par rapport à la tension
Voltage lags current: Tension en arrière par rapport au courant
True power: Puissance active
Reactive power: Puissance réactive
Apparent power: Puissance apparente
Power triangle: triangle des puissances
Power Factor: Facteur de puissance
Voltage Drop: Chute de tension
Boucherot theorem: Théorème de Boucherot

Basic Formulas

$$< v(t) >= \frac{1}{T}\int_0^T v(t).dt = 0$$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = \frac{V_{peak}}{\sqrt{2}}$$

$$|\underline{Z}| = \sqrt{R^2 + X^2} \qquad R = |\underline{Z}|\cos\varphi \quad X = |\underline{Z}|\sin\varphi$$



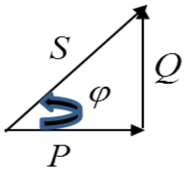
$$V = RI$$

$$\underline{V} = jL\omega\underline{I}$$

$$\underline{V} = -j\frac{1}{C\omega}\underline{I}$$

$\underline{S} = \underline{V} \times \underline{I}^{*} = P + jQ$

$|\underline{S}| = \sqrt{P^2 + Q^2}$ $\qquad P = |\underline{S}| \cos\varphi$ $\qquad Q = |\underline{S}| \sin\varphi$



$$\cos\varphi = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}$$

$$\underline{Z_{cond}} = (r_{cond} + jx_{cond})$$

$$\Delta V = r_{cond} I \cos\varphi + x_{cond} I \sin\varphi$$

$$P = \sum_{i=1}^{n} P_i$$

$$Q = \sum_{i=1}^{n} Q_i$$

$$\underline{S} = \sum_{i=1}^{n} \underline{S_i} = P = \sum_{i=1}^{n} P_i + jP = \sum_{i=1}^{n} Q_i$$

CHAPTER THREE

Three-Phase AC POWER

Learning Outcomes:

This chapter introduces the concepts and principles of the three-phase electrical supply, and the corresponding circuits. On completion you should be able to:

1. Describe the reasons for, and the generation of the three-phase supply.
2. Distinguish between star (3 and 4-wire) and delta connections.
3. State the relative advantages of three-phase systems compared with single-phase-systems.
4. Solve three-phase circuits in terms of phase and line quantities and the power developed in three-phase balanced loads.
5. Measure power dissipation in both balanced and unbalanced three-phase loads, using the
1, 2 and 3-wattmeter methods, and hence determine load power factor.

Introduction

- Three-phase AC power perform better (higher efficiency, lower maintenance, etc.) than single-phase.

Most electrical equipment in industrial premises operates on a three-phase supply. All industrial premises have a variety of single-phase and three-phase loads. It is essential to know how to connect them correctly.

Three-phase ac power is one of the most common forms of electric power distribution worldwide. Many countries use three-phase ac power for power distribution since it is simpler, cheaper, and more efficient than single-phase ac power.

Up till now, we have been talking only about single-phase AC power.
However, power companies generate and distribute three-phase power. Three-phase power is used in commercial and industrial applications that have higher power requirements than a typical residence.
Three-phase power, as shown in the following illustration. Each wave represents a phase and is offset by 120 electrical degrees from each of the two other phases

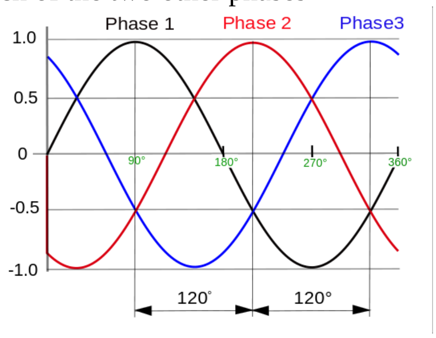


Figure 1. Three phase waveforms

Three Phase Systems

Now we consider the arrangement of three voltage sources illustrated in Figure 2. The three phase voltages are:

$$V_{L1} = V_L\sqrt{2}\sin(\omega t) \rightarrow V_{L2} = V_L\sqrt{2}\sin(\omega t - \frac{2\pi}{3}) \rightarrow V_{L3} = V_L\sqrt{2}\sin(\omega t - \frac{4\pi}{3})$$

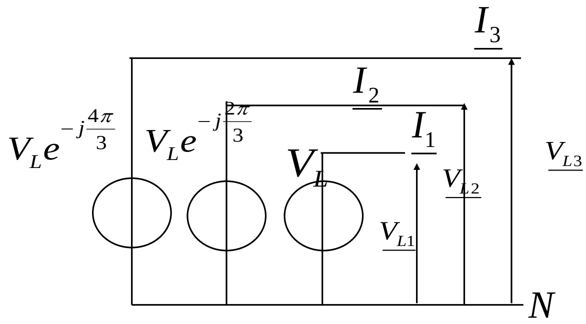


Figure 2 Three voltage sources

Star and delta configurations

The windings of a three-phase ac power source can be connected in either a **star configuration**, or a **delta configuration**

Star configuration Y (wye configuration)

The mains 400 / 230V supply is derived from a substation transformer. The transformer secondary windings are connected in **star** as shown in figure 1. The **star point** provides a neutral for the system. The star point is earthed by the Supply Authority.

Loads may be connected in either **line-to-neutral** or **line-to-line** configuration.

Note: The star point and neutral of the transformer secondary winding is earthed

Figure 3a illustrates a typical three-phase four-wire distribution system

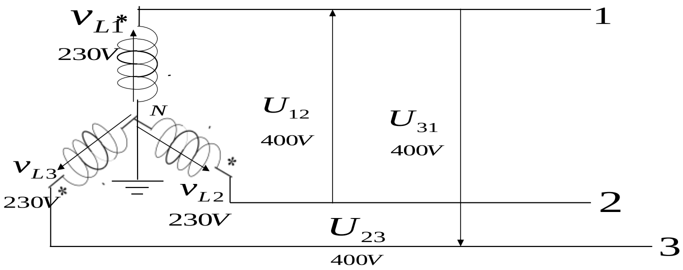


Figure 3a Three-phase four-wire distribution systems

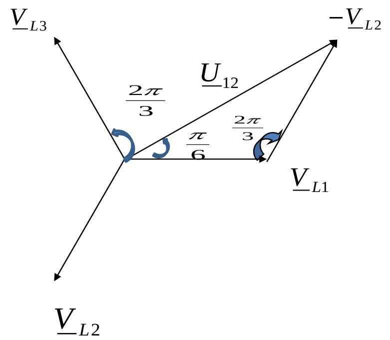


Figure 3b Phasor Diagram

Line Voltage $\underline{V}_L$

This is the voltage of **each phase** conductor with respect to the neutral conductor. It is illustrated in Figure 3a. The value of each phase voltage is **<u>230 Volts</u>** AC (RMS).

The three line voltages are: ($\underline{V}_{L1}, \underline{V}_{L2}, \underline{V}_{L3}$)

Line to line Voltage (U)

This is the voltage between any **two lines** (e.g. between L1 and L2, L1 and L3, L2 and L3). It is illustrated in Figure 3a. The value of each line to line voltage is **<u>400 Volts</u>** AC (RMS).

The three line to line voltages are: ($\underline{U}_{12}, \underline{U}_{23}, \underline{U}_{31}$)

Relationship of **line-to-line** and line voltages:

$$\underline{U} = \sqrt{3} V_L e^{j\frac{\pi}{6}} \quad U = 2V_L \cos\frac{\pi}{6} = 2V_L \frac{\sqrt{3}}{2} = V_L\sqrt{3} \qquad U = V_L\sqrt{3}$$

The phasor relationship of **line-to-neutral** and **line-to-line** voltages is shown in Figure3b. Two things should be noted about this relationship:

- The line-to-line voltage set has a magnitude that is larger than the line voltage by a factor of $\sqrt{3}$
- **Line-to-line** voltages are phase shifted by **30∘** ahead of **line-to-neutral** voltages.

It is seen from figure 1 that in the balanced system shown, the three phases are equal in magnitude and differs in phase angle by 120^0. The corresponding phasor diagram is shown in figure 3b.

Since the three phases are usually 120º out of phase, their **<u>phasor</u>** addition will be zero if the supply is balanced.

$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0 \qquad \underline{V}_{L1} + \underline{V}_{L2} + \underline{V}_{L3} = 0 \qquad \underline{U}_{12} + \underline{U}_{23} + \underline{U}_{31} = 0$

For a balanced **star** connected load with line to line voltage U and line current **I_L**,

$$V_L = \frac{U}{\sqrt{3}}; I_s = I_L$$

$$Z_s = \frac{V_L}{I_s}$$

$$\underline{S}_s = \sqrt{3} \times \underline{U}\,\underline{I}_L^*$$

$$P = \sqrt{3} U I_L \cos\varphi$$

$$Q = \sqrt{3} U I_L \sin\varphi$$

$$S = \sqrt{3} U I_L$$

Exercise 1

The peak value of the line to line voltage in all three phases is 563, 38 *V*. Calculate:

a) The effective line voltage (voltage between a phase and neutral line);

b) The effective line to line voltage that can be taken between two lines

Delta configuration D

Figure 4 shows that, in a delta-connected circuit, the three windings of the three-phase ac power source are connected one to another, forming a triangle.

The three line wires are connected to the three junction points of the circuit (points 1, 2, and 3 in Figure 4). There is no point to which a neutral wire can be connected in a three-phase delta-connected circuit. Thus, delta connected systems are typically three-wire systems.

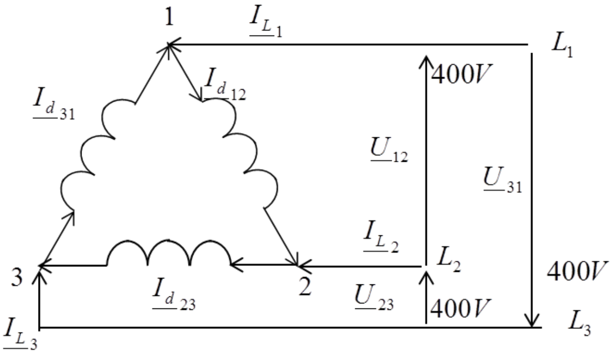


Figure 4a Delta-connected circuit

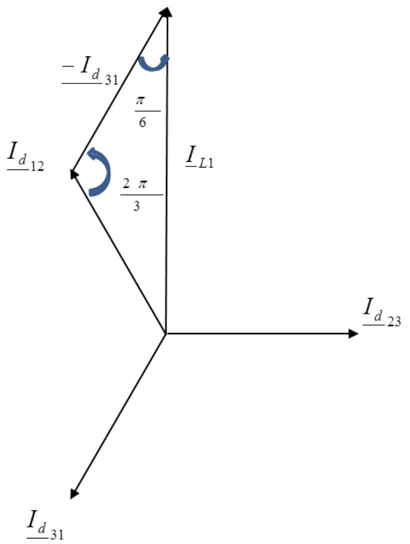


Figure 4b Phasor Diagram

Similarly in the case of a delta connected supply, the current in the line is √3 times the current in the delta.

Relationship of current in the line and the current in the delta:

$$\underline{I}_L = I_d e^{-j\frac{\pi}{6}} \qquad I_L = 2I_d \cos\frac{\pi}{6} = 2I_d \frac{\sqrt{3}}{2} = I_d\sqrt{3} \quad I_L = I_d\sqrt{3}$$

The star delta transformation

It can be shown, for a balanced load (using the star delta transformation or otherwise), that the equivalent delta connected impedance is **3 times** that of the star connected impedance. The phase angle of the impedance is the same in both cases. $Z_D = 3 \times Z_Y$

Exercise 2. Suppose it is necessary to build a resistive heater to deliver 6kW, to be made of three elements which may be connected in either wye or delta. Each of the three elements must dissipate 2000 W. Find R_Y and R_D

Three Phase Power

In the case of single phase, we learnt that the active power is given by

$P = VI \cos\varphi$

In the case of three phases, obviously this must apply for each of the three phases. Thus

$P = 3V_L I_L \cos\varphi$. The reactive power is given by: $Q = 3V_L I_L \sin\varphi$

The expression for apparent power is: $S = 3V_L I_L$ $\underline{S} = 3\underline{V}_L \times I_L^*$

For a balanced **delta** connected load with line to line voltage **U** and line current **IL**,

$$U; I_d = \frac{I_L}{\sqrt{3}}$$

$$Z_d = \frac{U}{I_d}$$

$$\underline{S}_s = \sqrt{3} \times U \times \underline{I}_L^*$$

Thus for a three phase system (in fact we do not even have to know whether it is a load or not, or whether it is star-connected or delta-connected)

$$P = \sqrt{3} U I_L \cos\varphi$$

$$Q = \sqrt{3} U I_L \sin\varphi$$

$$S = \sqrt{3} U I_L$$

Three-Phase Power Measurements

Three-wattmeter configuration

For a three-phase system, a single-phase wattmeter can be connected in each phase as shown in Figure 5. Using this three-wattmeter configuration, the total real power can be obtained by adding the three wattmeter readings. $P = P_1 + P_2 + P_3$

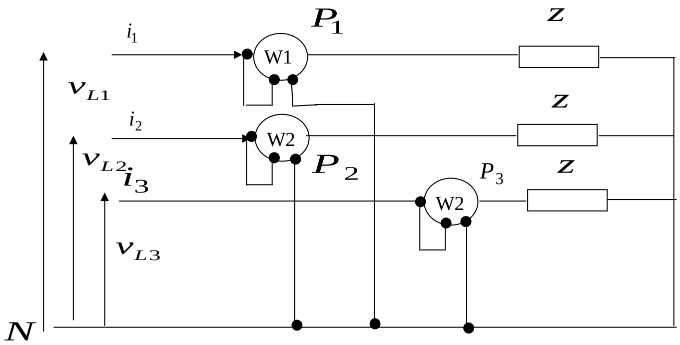


Figure 5. Three-wattmeter configuration

Two-wattmeter configuration

As it turns out, it is more effective to use two-wattmeter for measuring three-phase power. This two-wattmeter configuration is shown in Figure 6.

The two wattmeter readings are: $P = P_1^{'} + P_2^{'}$

It can be seen that: $Q = \sqrt{3}(P_1^{'} - P_2^{'})$

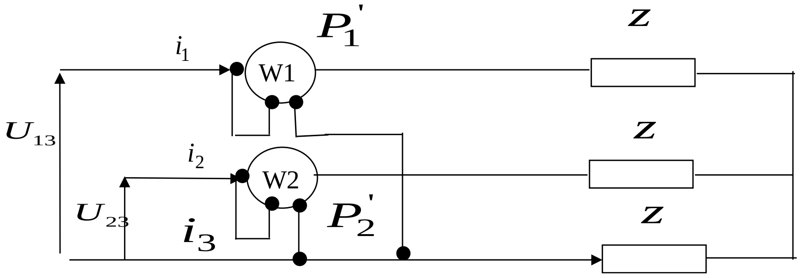


Figure 6 Two-wattmeter configuration

REVIEW QUESTIONS

1. In a balanced star-connected circuit, the

a. Line voltages and currents equal the load values.

b. Line to line voltage is 3 times greater than the line voltage.

c. Line to line voltage is 3 times smaller than the line current.

d. Line current is 3 times greater than the delta current.

2. In a balanced delta connected circuit, the

a. Line voltages and currents equal the load values.

b. Line current is 3 times smaller than the delta current.

c. Line current is $\sqrt{3}$ times greater than the delta current.

d. Line to line voltage is 3 times greater than the line voltage.

3. What is the line-to-neutral voltage in a balanced star-connected circuit when the line-to-line voltage is 400V?

a. 346V

b. 600V

c. 200V

d. 230V

4. What is the line current in a balanced delta-connected resistive load when the load current through each branch is 10A?

a. 27.3A

b. 17.3A

c. 11.6A

d. 5.8A

5. In a three-phase balanced circuit, the active power can be determined using two wattmeters connected according to the:

a. Single-phase wattmeter method

b. Three-phase wattmeter method

c. Two-wattmeter method

d. Apparent power method

6. The formula for total active power in a three-phase balanced circuit is:

a. $P_{active} = \sqrt{3}(V_L \times I_L \times \cos\varphi)$

b. $P_{active} = 3(U \times I_{line} \times \cos\varphi)$

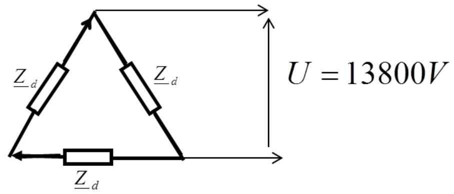
c. $P_{active} = 3(V_L \times I_L \times \cos\varphi)$

d. $P_{active} = \sqrt{3}(U \times I_L \times \cos\varphi)$

Exercise 3.

A circuit is set up as shown below. The frequency of the source is 50Hz. Given $\underline{Z} = (240 + j70)\Omega$.

1. Convert the load to Y and work with one line
2. Calculate *P, Q, S*



Keywords

Three Phase System: Système triphasé
Wye or star configuration: Couplage étoile
Delta configuration: Couplage triangle
Phase Voltage (VPh): Tension simple
Line Voltage (VL): Tension composée
Current in the line: Courants de ligne
Current in the delta: Courant polygonal
Phasor Diagram: Diagramme de Fresnel
Three Phase Power: Puissance triphasée
Three-wattmeter configuration: Méthode des trois wattmètres
Two-wattmeter configuration: Méthode des deux wattmètres

Basic Formulas

$U = V_L\sqrt{3}$

$I_L = I_d\sqrt{3}$

$P = \sqrt{3}UI_L\cos\varphi$

$Q = \sqrt{3}UI_L\sin\varphi$

$S = \sqrt{3}UI_L$

$\underline{S} = \sqrt{3} \times \underline{U} \times \underline{I}_L^* = (P + jQ)$

$P = P_1' + P_2'$

$Q = \sqrt{3}(P_1' - P_2')$

CHAPTER FOUR

Iron Core Inductor

Learning Outcomes:

By studying this chapter, you will learn:

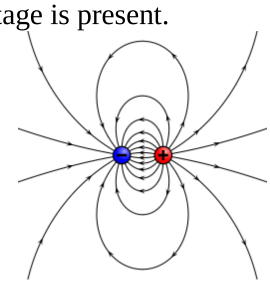
- How magnetic field lines are different from electric field lines.
- Some practical applications of magnetic fields in physics.
- How to analyze magnetic forces on current-carrying conductors.
- What Ampere's law is, and what it tells us about magnetic fields.
- How to use Ampere's law to calculate the magnetic field of symmetric current distributions

Introduction

This chapter will develop some basic tools for the analysis of magnetic field systems and will provide a brief introduction to the properties of practical magnetic materials. These results will then be applied to the analysis of transformers and rotating machines. So a careful study for this chapter is recommended to fully understand the next chapters.

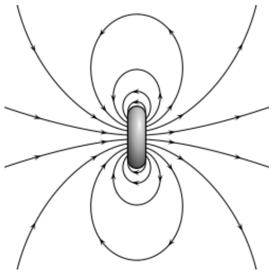
Electromagnetism describes the relationship between electricity and magnetism. It is essentially the foundation for all of electrical engineering. We use electromagnets to generate electricity, store memory on our computers, generate pictures on a television screen, diagnose illnesses, and in just about every other aspect of our lives that depends on electricity.

An electric field occurs wherever a voltage is present.



Electric dipole field lines

Magnetic fields are created whenever there is a flow of electric current



Magnetic dipole field lines

Comparison chart

	Electric Field	Magnetic Field
Nature	Created around electric charge	Created around moving electric charge and magnets
Units	volts per meter	Ampere/meter
Force	Proportional to the electric charge	Proportional to charge and speed of electric charge
Movement In Electromagnetic field	Perpendicular to the magnetic field	Perpendicular to the electric field
Electromagnetic Field	Generates VARS (Capacitive)	Absorbs VARS (Inductive)
Pole	Monopole or Dipole	Dipole
Nature	Created around electric charge	Created around moving electric charge and magnets

The magnetic field

Magnetic fields are the fundamental mechanism by which energy is converted from one form to another in motors, generators and transformers.
First, we are going to look at the basic principle – A current-carrying wire produces a magnetic field in the area around it.
Production of a Magnetic Field (Reminders)
Ampere's Law – the basic law governing the production of a magnetic field by a current:

$$\oint Hdl = I$$

Where **H** is the magnetic field intensity produced by the current **I** and **dl** is a differential element of length along the path of integration. H is measured in Ampere-turns per meter.

In this sense, H (Ampere turns per meter) is known as the effort required to induce a magnetic field. The strength of the magnetic field flux produced in the core also depends on the material of the core. Thus,

$$B = \mu \times H$$

B = magnetic flux density (Tesla (T))
μ= magnetic permeability of material (Henrys per meter)
H = magnetic field intensity (ampere-turns per meter)
The constant **μ** may be further expanded to include relative permeability which can be defined as below:

$$\mu = \mu_0 \times \mu_r$$

Where: μ_0 permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ and μ_r is the relative permeability
The value of relative permeability is dependent upon the type of material used. The higher the amount permeability, the higher the amount of flux induced in the core

The symbols for different inductors are provided in Figure 1.

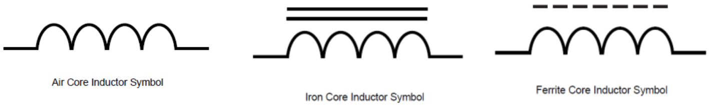


Figure 1. Inductor (coil) symbols

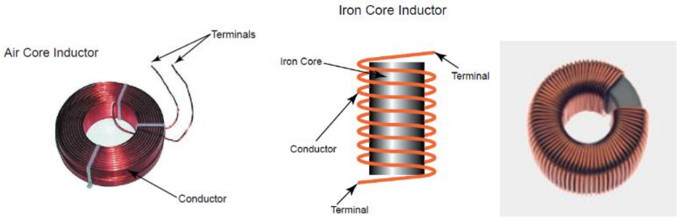


Figure 2. Inductor (coil) images

The magnetic system

A magnetic core is a piece of magnetic material with a high permeability used to confine and guide magnetic fields in electrical, electromechanical and magnetic devices.

Suppose we were to wrap a coil of insulated wire around a loop of ferromagnetic material and energize this coil with an AC voltage source:

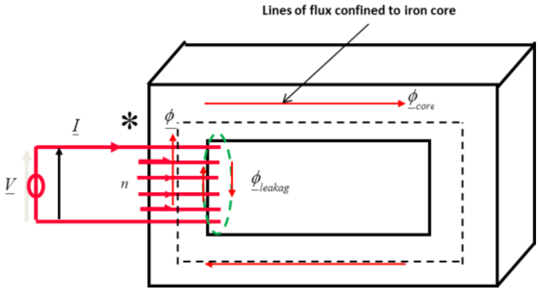


Figure 3. Iron core inductor.

I is the current in amperes

Φ is the total flux in weber

ϕ_{core} is the flux in the core in weber

$\phi_{leakage}$ is the leakage flux in weber

l_{core}: Mean magnetic path length in meters

Where n is the number of turns

A wire will generate a magnetic field when current flows through it. Any change in the current through an inductor creates a changing flux, inducing a voltage across the inductor. By Faraday's law of induction, the voltage induced by any change in magnetic flux through the circuit is:

The induced voltage of an inductor can be related to the flux density by

$\underline{V} = j\omega\underline{\Phi}$ $\quad \omega = 2\pi f$

The **total** flux is found from: $\Phi = B \times S = \phi_{core} + \phi_{leakage}$

B is the maximum flux density in tesla, n is the number of turns, f is the frequency in hertz, and S is the cross-sectional area of the core in square meters. This important equation is derived from Faraday’s law.

$$V_{RMS} = n \times \frac{\Phi_{max}}{\sqrt{2}} \times \omega = n \times \frac{B_{max}}{\sqrt{2}} \times S \times 2\pi \times f = 4{,}44 \times B \times n \times f \times S$$

B is the magnetic flux density and H is the magnetizing force that generated the flux.

Permeability: Figure of merit of a particular magnetic material representing the ease of producing a magnetic flux for a given input. $\mu = \frac{B}{H}$

H, the Magnetizing Force that produces the Flux, is measured in Amps per Meter

Permeability μ is in (H/m)

Modeling the iron core inductor

The Magnetic equivalent circuit is:

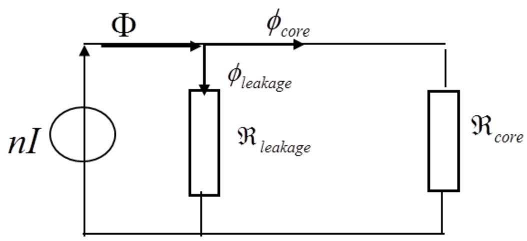


Figure 4 magnetic equivalent circuit

When the Kirchhoff's current law is applied to the circuit, we have: $\Phi = \phi_{core} + \phi_{leakage}$

(In any real circuit, there is leakage flux $\phi_{leakage}$, but we ignore this for now)

For simplicity, the effect of leakage flux is **negligible** and the flux distribution is assumed to be as in figure 5. The Magnetic equivalent circuit becomes:

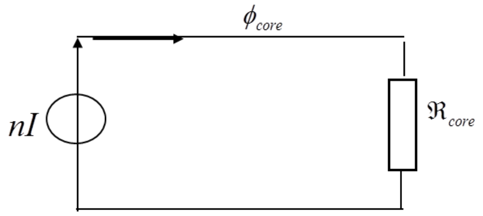


Figure 5 Magnetic equivalent circuit

Then, the following relation can be obtained: $n \times I = \Re \times \phi_{core}$

From Ampere's law, we have: $n \times I = H \times l_{core}$

SELF-INDUCTANCE

The ability of a coil to oppose any change in current is a measure of the self-inductance L of the coil. Inductance is measured in henry (H). The following relation can be obtained:

$$n \times I = \Re \times \phi_{core}$$

$$n \times \phi_{core} = L \times I \rightarrow \phi_{core} = \frac{L \times I}{n}$$

$$n \times I = \Re \times \frac{L \times I}{n}$$

$$L = \frac{n^2}{\Re} = \mu \times S \frac{n^2}{l_{core}}$$

Reluctance depends on the dimensions of the core as well as its materials:

$$\Re = \frac{l_{core}}{\mu \times S}$$

We can describe **units** for reluctance $\Re$ as amp-turns per weber (At / Wb)

Inductive reactance

The effective resistance of the coil in an AC circuit is measured by a quantity called the **inductive reactance** X_L:

$$X_L = L\omega = 2\pi fL$$

When *f* is in hertz and *L* is in henry, the unit of X_L is the ohm. The inductive reactance *increases* with increasing frequency and increasing inductance.

Analogy between magnetic circuit and electric circuit

Some corresponding quantities in electric and magnetic circuit are listed as below.

Electric quantities	Magnetic quantities
Current I	Magnetic flux Φ
Current density J	Magnetic flux density B
Conductivity σ	Permeability μ
Electromotive force = resistance ×I	Magnetomotive force = reluctance ×Φ
Electric field intensity E	Magnetic field intensity H
Conductance = 1/resistance	Permeance = 1/reluctance
Resistance = l/σ×S	Reluctance = l/μS

Table1: Analogy between magnetic circuit and electric circuit

Electrical Equivalent circuit for the practical iron-core

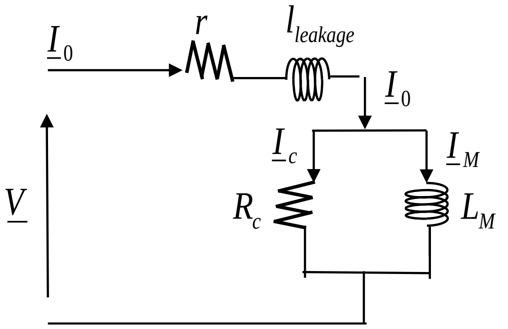


Figure 6 Electrical Equivalent circuit for the practical iron-core

When the Kirchhoff's current law is applied to the circuit, we have: $\underline{I}_0 = \underline{I}_c + \underline{I}_M$

r is the dc resistance of the winding

R_c represents the hysteresis and eddy current losses

L_m (magnetizing inductance) is the inductance associated with the magnetization of the core

$l_{leakage}$ is the inductance associated with the leakage flux

I_0 excitation current

I_M is magnetizing current

I_c is core loss current

Core Losses

The total core losses are made up of three main components:

1. **Copper losses**: The power lost by current flowing through the winding. The power loss is equal to the square of the current multiplied by the resistance of the wire (I^2 R). This power loss is transferred into heat.
2. **Eddy current losses**: Eddy current losses are present in the magnetic core:
   Laminated cores:
   Cores constructed by stacking multiple laminations on top of each other. Each lamination has an insulated surface which is commonly an oxide finish. Laminated cores are used in a wide variety of transformer applications

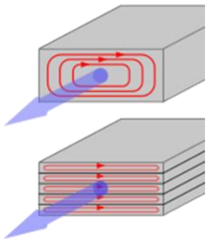


Figure 7 Laminated cores

Power dissipation of eddy currents: p_{core}

$$p_{core} = qM \frac{f}{50} B_{\max}^2$$

$q(W/kg)$: is the constant of the materiel

$M(kg)$: is the mass of the circuit

$f(Hz)$: is the frequency

$B_{\max}(T)$: is the peak magnetic field

3. **Hysteresis losses**
   Each time the magnetic field is reversed, a small amount of energy is lost due to hysteresis within the core

Saturation (magnetic) figure 8
Saturation is most clearly seen in the magnetization curve (also called BH curve or hysteresis curve) of a substance, as a bending to the right of the curve (see graph at right). As the H field increases, the B field approaches a maximum value asymptotically, the saturation level for the substance.

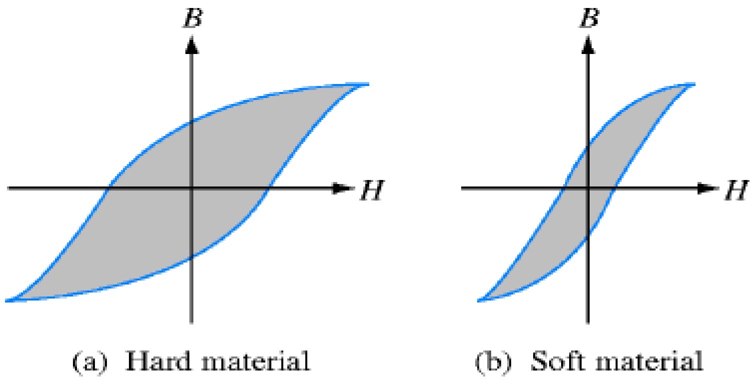


Figure 8 Magnetic core BH characteristic

The relation between the magnetizing field H and the magnetic field B can also be expressed as: $B = \mu H$. The permeability of ferromagnetic materials is not constant, but depends on H. In materials with saturable nonlinearity the relative permeability increases with H to a maximum, then as it approaches saturation inverts and decreases toward one.
In general, permeability is not a constant. Relative permeability, sometimes denoted by the symbol μ_r, is the ratio of the permeability of a specific medium to the permeability of free space, μ_0: $\mu_r = \frac{\mu}{\mu_0}$. Where $\mu_0 = 4\pi \times 10^{-7}$ (H/m)
For a **nonlinear magnetic system (figure 9)**, however, the magnetic field H_c can only be calculated by the above definition with all coils switched on.

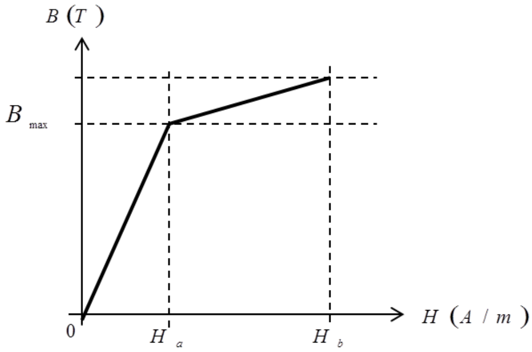


Figure 9 Magnetic core BH characteristic

$$H_c = \frac{1}{\mu_0 \times \mu_r} \int_a^b dB$$

Magnetic system with and Air Gap

Figure 10 shows a simple magnetic circuit with an air gap

M. Nasser

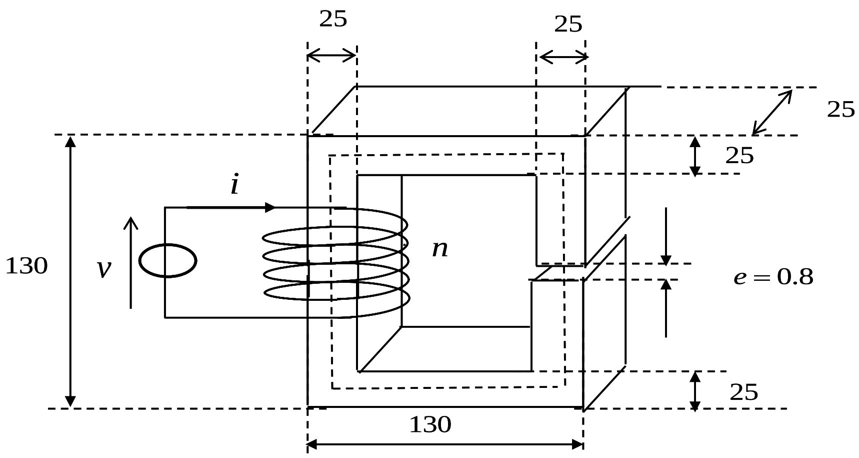


Figure10 Magnetic system with an air gap.

The Magnetic equivalent circuit (figure 11) is:

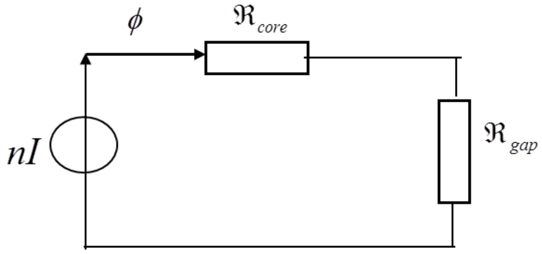


Figure11. Magnetic equivalent circuit

Applying the Ampère's circuital law, one gets: $nI = H_c l_c + H_g l_g$

Where the subscript **c** refers to the core and **g** to the air gap. The path length $\mathbf{l_c}$ in the core can be taken to be the length measured along the center of the cross section of the core. $\mathbf{H_c}$ and **Hg** can be written in terms of the magnetic flux as

$$H_c = \frac{B_c}{\mu_c} = \frac{\phi_c}{\mu_c \times S_c}$$

$$H_g = \frac{B_g}{\mu_g} = \frac{\phi_g}{\mu_0 \times S_g}$$

According to Gauss's law of magnetism, the net outward flux of B through any closed surface must be equal to zero. Hence, the flux of B must be the same over any cross section of the magnetic circuit and $\phi_c = \phi_g$

Exercise 1

Give the Magnetic equivalent circuit and find the value of the current I required to establishing a flux density of 1.4 T in the magnetic system as shown in figure 12. Leakage flux is ignored. Given $n = 1200 turns; \mu_0 = 4\pi \times 10^{-7} (H/m)$.

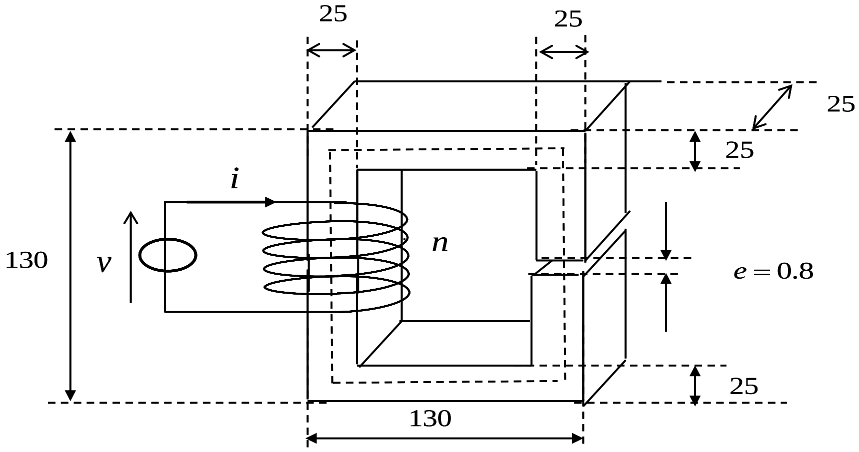


Figure12 Magnetic system with an air gap.

The magnetization date for the material is as follows:

B (T)	0	0,5	0,8	1	1,2	1,4	1,6	1,8
H (A/m)	0	220	490	760	1300	2450	4700	11500

Table 1. B-H for selected materials

Keywords

Electromagnetism: Electromagnétisme
The magnetic system: Système magnétique
Iron Core Inductor: Bobine à noyau de fer
Coil: Bobine
Core: Noyau
Air gap: Entrefer
Flux: Flux
Leakage flux: Flux de fuite
Inductance: Inductance
Reactance: Réactance
Reluctance: Reluctance
Magnetizing inductance: Inductance de magnétisation
Magnetizing reactance: reactance de magnétisation
Magnetic flux density: densité du flux magnétique
Magnetizing force: force magnétomotrice
Permeability: Perméabilité
Mean magnetic path length: longueur de la ligne moyenne du flux magnétique
Number of turns: Nombre de spires
Magnetic Equivalent Circuit: Schéma magnétique équivalent
Electrical Equivalent circuit: Schéma électrique équivalent
Hysteresis losses: Pertes par hystérisis
Eddy or core losses: Pertes fer ou par courants de Foucauld
Copper or Joule losses: Pertes cuivre ou pertes joules
Excitation current: Courant d'excitation
Magnetizing current: Courant de magnétisation
Core loss current: Courants de Foucauld
Saturation: Saturation
Nonlinear magnetic system: Système magnétique non linéaire

Basic Formulas

$\Phi = L \times i$

$\Phi = B \times S$

$\mu = \frac{B}{H} = \mu_r \times \mu_0$

$$\mu_r = \frac{\mu}{\mu_0}$$

$$\boldsymbol{n \times I = \Re \times \Phi}$$

$$\boldsymbol{n \times I = H \times l_{core}}$$

$$L = \frac{n^2}{\Re} = \mu \times S \frac{n^2}{l_{core}}$$

$$\underline{\boldsymbol{I}}_0 = \underline{\boldsymbol{I}}_c + \underline{\boldsymbol{I}}_M$$

$$\mu_0 = 4\pi \times 10^{-7}$$

$$H_c = \frac{1}{\mu_0 \times \mu_r} \int_a^b dB$$

$$NI = H_c l_c + H_g l_g$$

$$H_c = \frac{B_c}{\mu_c} = \frac{\phi_{core}}{\mu_c \times S}$$

$$H_g = \frac{B_g}{\mu_g} = \frac{\phi_g}{\mu_0 \times S_g}$$

CHAPTER FIVE

A single-phase transformer

Learning Outcomes:

After successfully studying this chapter, students will be able to:

- Explain the production of eddy current losses within the core, and the constructional features that reduce them
- Calculate applied voltage and induced voltage.
- Draw and label both the full equivalent circuit and the simplified equivalent transformer circuit
- Calculate equivalent resistance and reactance
- Calculate equivalent circuit parameters from open and short circuit test results.
- Calculate transformer efficiency for full load and half load, each with different power factors

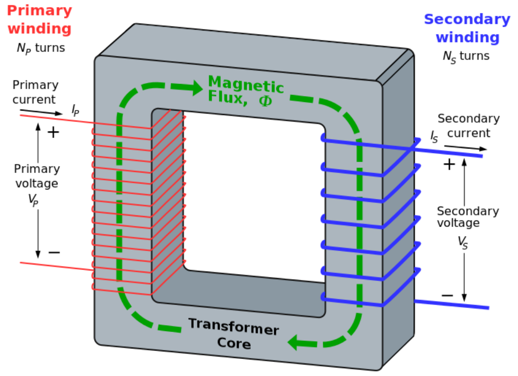
Introduction

A transformer is an electrical device that transfers energy between two circuits through electromagnetic induction. For example, the transfer of electricity efficiently over a long transmission line requires the use of high voltages. At the receiving end where the electricity is used, the high voltage has to be reduced to the levels required by the consumer.

A transformer used to increase the voltage is called a "**step up**" transformer, while that used to decrease the voltage is called a "**step down**" transformer.

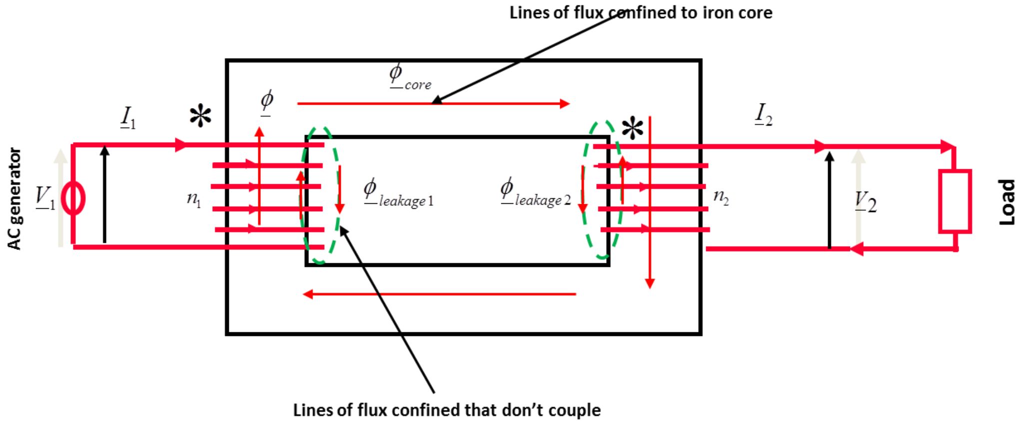
A transformer consists of **two windings** of wire that are wound around a **common core** to provide tight electromagnetic coupling between the windings. The core material is often a laminated iron core. The coil that **receives** the electrical input energy is referred to as **the primary** winding, while the output coil is called **the secondary** winding.

(A transformer cannot operate with direct current)



Basic principles

The functioning of a transformer is based on two principles of the laws of electromagnetic induction: An electric current through a conductor, such as a wire, produces a magnetic field surrounding the wire, and a changing magnetic field in the vicinity of a wire induces a voltage across the ends of that wire.



Ideal transformer

The assumptions to characterize the ideal transformer are:

- The windings of the transformer have no resistance. Thus, there is no copper loss in the winding, and hence no voltage drop.
- Flux is confined within the magnetic core. Therefore, it is the same flux that links the input and output windings.
- Permeability of the core is infinitely high which implies that net mmf (amp-turns) must be zero (otherwise there would be infinite flux) hence $I_1 n_1 - I_2 n_2 = 0$.
- The transformer core does not suffer magnetic hysteresis or eddy currents, which cause inductive loss.

If the secondary winding of an ideal transformer has no load, no current flows in the primary winding.

The ideal transformer induces secondary voltage V_2 as a proportion of the primary voltage V_1 and respective winding turns n_1, n_2 as given by the equation:

$$\frac{V_2}{V_1} = \frac{n_2}{n_1} = m$$

m : is the winding **turns ratio**, the value of these ratios being respectively higher and lower than unity for step-up and step-down transformers
V_1 : designates **input** voltage,
V_2 : designates **output** voltage

Real transformer

The ideal transformer model neglects the **core losses**, collectively called magnetizing current losses, consist of:

- Hysteresis losses due to nonlinear application of the voltage applied in the transformer core, and
- Eddy current losses due to joule heating in the core that are proportional to the square of the transformer's applied voltage.

Whereas windings in the ideal model have no impedance, the windings in a real transformer have finite non-zero impedances in the form of:

- Joule losses due to resistance in the primary and secondary windings
- Leakage flux that escapes from the core and passes through one winding only resulting in primary and secondary reactive impedance.

If a voltage is applied across the primary terminals of a real transformer while the secondary winding is open without load, the real transformer must be viewed as a simple inductor with an impedance Z.

Equivalent circuit of real transformer

Winding joule losses and leakage reactance are represented by the following series loop impedances of the model. **Core losses** and magnetizing reactance are represented by the following shunt leg impedances of the model.

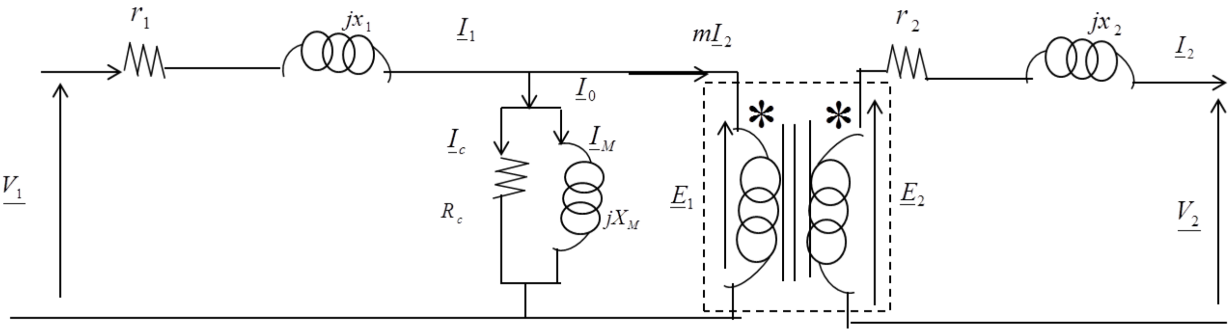


Figure 1 Equivalent circuit of real transformer

The following legend applies to the transformer formulas:

- Resistance and reactance of primary winding: r_1, x_1
- Resistance and reactance of secondary winding: r_2, x_2.
- Primary induced voltage: E_1
- Secondary induced voltage: E_2
- Primary terminal voltage: V_1
- Secondary terminal voltage: V_2
- Primary current: I_1
- Secondary current: I_2
- No load primary current: I_0
- Magnetizing current: I_M
- Current accounting for the core losses: I_C
- Core or iron losses: R_C
- Magnetizing reactance: X_M.

R_C and X_M are collectively termed the magnetizing branch of the model.

Phenomena not covered in the equivalent circuit

- Saturation
- Inrush current
- Sinusoidal exciting current

Approximate transformer equivalent circuit

- Transferring impedances through a transformer.

To transfer impedances of either side then the square of the Turns ratio is used. Figure 2 shows the Equivalent circuit when primary impedance is transferred to secondary side.

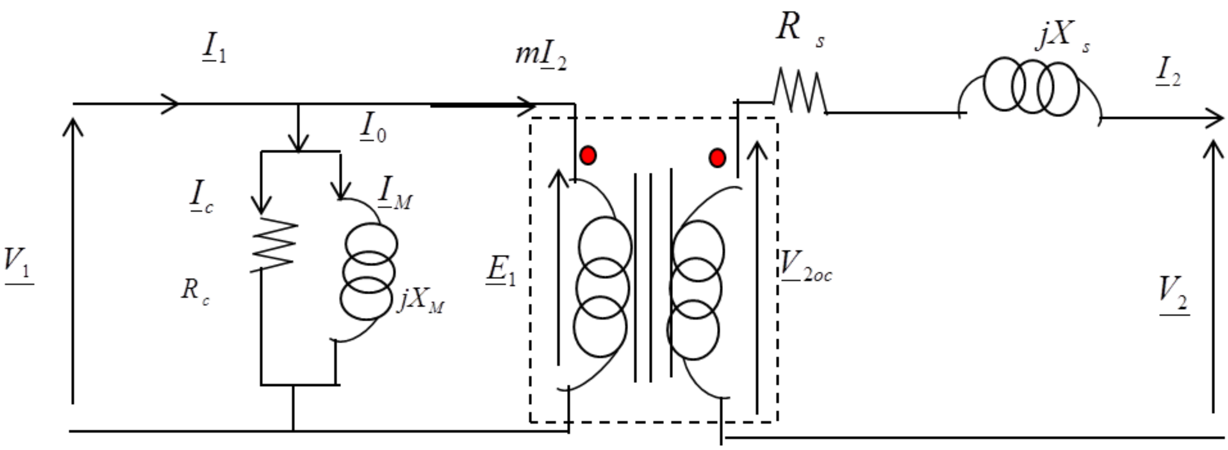


Figure 2 Approximate transformer equivalent circuit

$$R_s = r_2 + m^2 r_1$$
$$X_s = x_2 + m^2 x_1$$
$$Z_s = R_s + jX_s$$

R_s: Equivalent resistance transferred to secondary side.

X_s: Equivalent reactance transferred to secondary side.

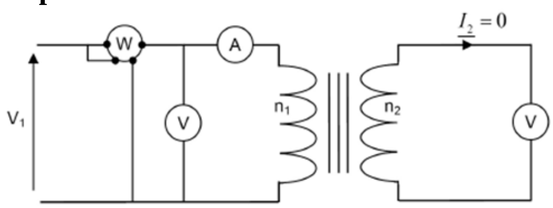
Z_s: Equivalent impedance is transferred to secondary side.

Determining transformer parameters

Approximate transformer equivalent circuit impedance and transformer ratio parameters can be derived from the following tests: open-circuit test, short-circuit test, and transformer ratio test.

1. Open circuit test

- Performed at **rated voltage**
- Determines **shunt components**



$$m = \frac{n_2}{n_1} = \frac{V_{2oc}}{V_1}$$

$$R_C = \frac{V_1^2}{P_{oc}}$$

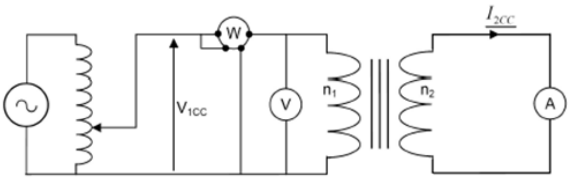
$$X_M = \frac{V_1^2}{Q_M}$$

$$Q_M = \sqrt{S_{1oc}^2 - P_{OC}^2}$$

$$S_{1oc} = V_1 I_{0c}$$

2. Short circuit test

- Performed at **rated current**
- Determines **series components**



$$Z_s = \frac{V_{2sc}}{I_{2sc}}$$

$$R_s = \frac{P_{sc}}{I_{2sc}^2}$$

$$X_s = \frac{Q_{sc}}{I_{2sc}^2} = \sqrt{Z_s^2 - R_s^2}$$

$$Q_{sc} = \sqrt{S_{1sc}^2 - P_{sc}^2}$$

$$S_{1sc} = V_{1sc} I_{1sc}$$

Transformer Losses

Most of the electrical energy provided to the primary of a transformer is transferred to the secondary. Some energy, however, is lost in heat in the wiring or the core. Some losses in the core can be **reduced** by building the core of a number of flat sections called **laminations**.

Winding joule losses p_{Cu}

Current flowing through winding conductors causes joule heating.

Hysteresis losses p_H

Each time the magnetic field is reversed, a small amount of energy is lost due to hysteresis within the core.

Eddy current losses p_{core}

Eddy currents circulate within the core in a plane normal to the flux, and are responsible for resistive heating of the core material.

Eddy current losses can be reduced by making the core of a stack of plates electrically insulated from each other, rather than a solid block; all transformers operating at low frequencies use **laminated** or similar cores.

Voltage Drop Calculations

Ohm's Law, expressed in phasor form for AC circuits, and gives the basic relationship for voltage drop and the load current.

$$\Delta V = R_s I_2 \cos\varphi_2 + X_s I_2 \sin\varphi_2$$

$$\underline{\Delta V} = \underline{Z_s} \times \underline{I_2}$$

$$\underline{Z_s} = (R_s + jX_s)$$

$\underline{\Delta V}$ is the voltage drop, in **volts**

$\underline{I_2}$ is the load current in **amperes**

$\underline{Z_s}$ is the conductor or equipment impedance, in **ohms**

Transformer Formulas

There are a number of useful formulas for calculating, voltage, current, and the number of turns between the primary and secondary of a transformer. These formulas can be used with either step-up or step-down transformers.

To find voltage: $V_{2oc} = \frac{I_1}{I_2} \times V_1$

For example, if a transformer has a primary voltage of 240 V, a primary current of 5 A and a secondary current of 10 A, the secondary voltage can be calculated as shown below:

$$V_{2oc} = \frac{I_1}{I_2} \times V_1 = \frac{5 \times 240}{10} = 120V$$

To find current: $I_2 = \frac{I_1}{V_{2oc}} \times V_1$

To find the number of coil turns: $n_2 = \frac{I_1}{I_2} \times n_1 = \frac{V_{2oc}}{V_1} \times n_1$ $\quad n_1 = \frac{I_2}{I_1} \times n_2 = \frac{V_1}{V_{2oc}} \times n_2$

The turns ratio of the transformer is: $m = \frac{n_2}{n_1} = \frac{V_{2oc}}{V_1} = \frac{I_1}{I_2}$

The transformer apparent power **rating** is: $S_r = V_{1r} \times I_{1r} = V_{2oc} \times I_{2r}$

The kVA **rating** determines the current and voltage a transformer can deliver to its load without overheating

Transformer Ratings

Transformers are rated for the amount of apparent power they can provide. Because values of apparent power are often large, the transformer apparent power rating is frequently given in kVA (kilovolt-amperes).

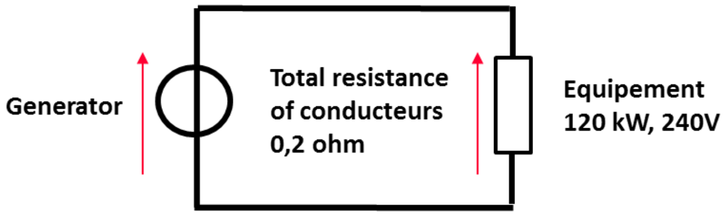
Condition for Maximum Efficiency

Iron Loss = Copper Loss

$$p_{core} = p_{Cu} = R_s I_2^2$$

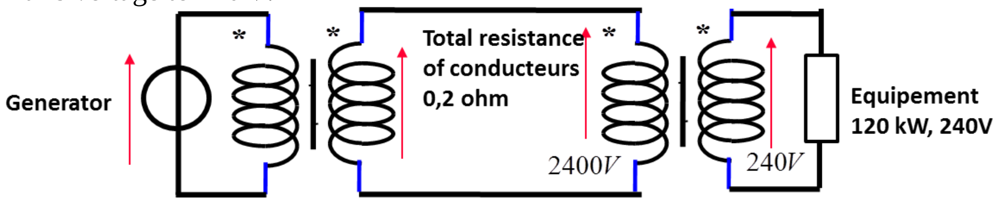
The value of Output current (I_2) on which Maximum efficiency can be gained is: $I_2 = \sqrt{\frac{p_{core} = p_{Cu}}{R_s}}$

Exercise 1. A farmer installs a private hydroelectric generator to provide power for equipment rated at 120 *kW* 240 *V* AC. The generator is connected to the equipment by two conductors which have a total resistance of $0{,}2\Omega$ -. The system is shown schematically in the figure below.



a) The equipment is operating at its rate power. Calculate:
(1) The power loss in the cables;
(2) The voltage which must be developed by the generator;
(3) The efficiency of the transmission system.

b) An engineer suggest that the farmer uses a transformer to convert the generator output to give a 2400 *V* at the end of the transmission line, as shown in the next figure. A second transformer is to be used to step down this voltage to 240 *V*.



(4) Explain briefly how a transformer makes use of electromagnetic induction to produce an output voltage several times bigger than the input voltage.
(5) The transformer is 100 % efficient. Calculate the power loss in the new transmission system.

Keywords

Transformer: Transformateur

Ideal transformer: Transformateur idéal

Real transformer: Transformateur réel

Step up transformer: Transformateur élévateur

Step down transformer: Transformateur abaisseur

Input voltage: Tension d'entrée

Output voltage: Tension de sortie

Turns ratio: Rapport de transformation

The primary: Le primaire

The secondary: Le secondaire

Electromagnetic induction: Induction électromagnétique

Wire: Bobine

Core: noyau

Equivalent circuit of real transformer: Schéma equivalent d'un Transformateur réel

Approximate transformer equivalent circuit: Schéma de Kapp d'un Transformateur

Resistance and reactance of primary winding: r_1, x_1: Résistance et reactance du primaire

Resistance and reactance of secondary winding: r_2, x_2: Résistance et reactance du secondaire

Primary induced voltage: E_1: Tension induite au primaire

Secondary induced voltage: E_2: Tension induite au secondaire

Primary terminal voltage: V_1: Tension au primaire

Secondary terminal voltage: V_2: Tension au secondaire

Primary current: I_1: Courant au primaire

Secondary current: I_2: Courant au secondaire

No load primary current: I_0: Courant à vide au primaire

Magnetizing current: I_M: Courant de magnétisation

Current accounting for the core losses: I_C: Courant de pertes fer

Core or iron losses: R_C: Résistance de fer

Magnetizing reactance: X_M: Réactance de fer

R_s : Equivalent resistance transferred to secondary side.
X_s : Equivalent reactance transferred to secondary side.
Z_s : Equivalent impedance is transferred to secondary side.
Saturation: Saturation

Inrush current: Courant de mise sous tension

Sinusoidal exciting current: Courant d'excitation sinusoïdal

Open circuit test: Essai à vide

Short circuit test: Essai en court-circuit

Transformer losses: Pertes du transformateur

Winding joule losses: Pertes joules

Hysteresis losses: Pertes par hystérisis

Eddy current losses: Pertes par courants de Foucauld

Transformer Ratings: Grandeurs nominales du transformateur

Basic Formulas

$$R_s = r_2 + m^2 r_1$$

$$X_s = x_2 + m^2 x_1$$

$$m = \frac{n_2}{n_1} = \frac{V_{2oc}}{V_1}$$

$$R_C = \frac{V_1^2}{P_{oc}}$$

$$X_M = \frac{V_1^2}{Q_M}$$

$$Q_M = \sqrt{S_{1oc}^2 - P_{OC}^2}$$

$$S_{1oc} = V_1 I_{0C}$$

$$Z_s = \frac{V_{2sc}}{I_{2sc}}$$

$$R_s = \frac{P_{sc}}{I_{2sc}^2}$$

$$X_s = \frac{Q_{sc}}{I_{2sc}^2} = \sqrt{Z_s^2 - R_s^2}$$

$$Q_{sc} = \sqrt{S_{1sc}^2 - P_{sc}^2}$$

$$S_{1sc} = V_{1sc} I_{1sc}$$

$$m = \frac{n_2}{n_1} = \frac{V_{2oc}}{V_1} = \frac{I_1}{I_2}$$

$$S_r = V_{1r} \times I_{1r} = V_{2oc} \times I_{2r}$$

$$P_1 = P_2 + P_{oc} + R_s \times I_2^2$$

$$P_2 = V_2 I_2 \cos\varphi_2$$

$$\eta = \frac{P_2}{P_2 + P_{oc} + R_s \times I_2^2}$$

CHAPTER 6

Three-Phase Transformers

Learning Outcomes:

By studying this chapter, you will learn:

- Star and delta winding connection
- Transformers – star-star, delta-delta, star-delta, delta-star; values – primary and secondary turns, voltage, current, VA.
- The relationship between line and phase values of voltage, current, and volt-amps (VA) are stated for star and delta winding configurations.
- Transformer efficiencies are calculated from given data for different values of secondary load
- Measurements are made according to industry practice

Introduction

For reasons of efficiency and economy in the use of copper, the generation, transmission, and distribution of electric power takes place on a three-phase basis rather than single-phase. Three-phase transformers are required to step-up or step-down voltages in the various stages of power transmission. A three-phase transformer can be built in one of two ways: by suitably connecting a bank of three identical single-phase transformers or by constructing a three-phase transformer on a common magnetic core. The single unit three-phase transformer consists of a three-legged construction with each leg carrying the primary and secondary coils of one phase as shown in Figure 1

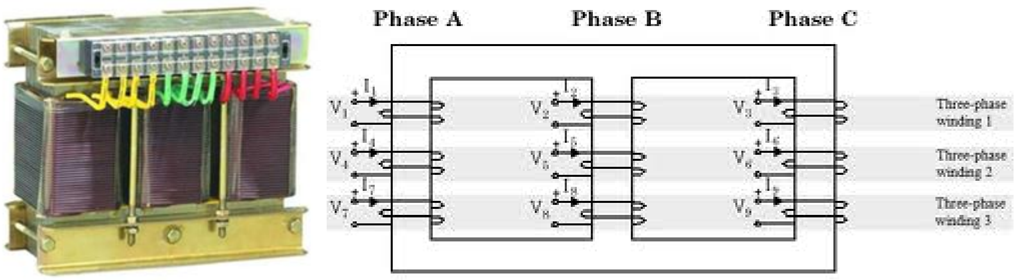


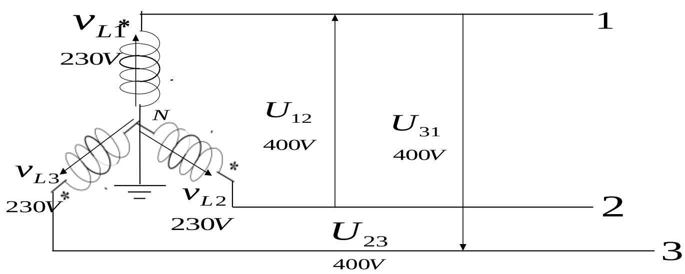
Figure 1. The single unit three-phase transformer

Any type of three-phase connection can be carried out on both sides of the transformer bank, such as Y/Y, Y/Δ, Δ/Y and Δ / Δ. Each transformer carries one third of the three-phase load under balanced conditions.

Star Connections

The wye connection is also known as a star connection. Three coils are connected to form a "Y" shape. The wye transformer secondary has four leads, three phase leads and one neutral lead. The voltage V across any phase (line-to-neutral) will always be less than the line-to-line U voltage.

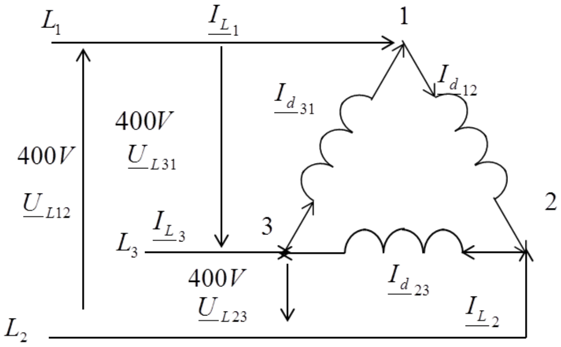
The major benefit of using a Y-connected winding in a transformer is that it provides a neutral point.



Delta Connections

Delta transformers are schematically drawn in a triangle. The voltages across each winding of the delta triangle represent one phase of a three phase system. The voltage is always the same between any two wires. All three phases are used to supply three phase loads.

Mainly, Δ-connected windings are used to suppress third harmonic currents in transformers; this third harmonic component may produce undesirable effects in three-phase transformer operation, particularly in the case of the wye-wye connection.
Therefore, Δ-windings need more expensive insulation as compared to Y-windings. The secondary of a delta transformer is illustrated in the figure below.
For simplicity, the secondary is not shown in this example. Just as with a single-phase transformer, the secondary voltage depends on both the primary voltage and the turns ratio.



When current is the same in all three coils, it is said to be balanced. In each phase, current has two paths to follow. For example, current flowing from L1 to the connection point at the top of the delta can flow down through one coil to L2, and down through another coil to L3.

Harmonics in Transformer

Harmonics in transformer occur due to the effect of saturation and Hysteresis which are to produce non-sinusoidal current if the applied voltage is sinusoidal.
Upon saturation, the flux waveform is flat topped and contains mainly 3rd harmonic component.

Effects of harmonic currents

(i) Additional RI^2 losses due to circulating currents.
(ii) Increased iron loss in core.
(iii) Magnetic interference with protective gear and communication circuits.

Effects of harmonic voltages

(i) Increased dielectric stress.
(ii) Electric field interference with communication circuit.
(ii) Harmonic resonance may occur between the inductance of transformer windings and the capacitance of a feeder to which it is connected.

Symbol designation

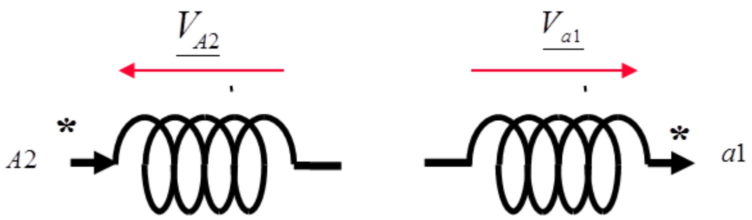
The vector group provides a simple way of indicating how the internal connections of a transformer are arranged. The vector group is indicated by a code consisting of two letters, followed by one or two numeric digits. The letters indicate the winding configuration as follows:

- D or d: Delta winding,
- Y or y: Wye winding, (also called a star).

A, B, C: capital letters indicates primary.
a, b, c: small letters indicates secondary

Transformer Polarity:

When we speak "the polarity" of transformer windings, we are identifying all of the terminals that are the same polarity at any instant of time. "Polarity marks" are employed to identify these terminals. These marks may be black dots, crosses, numerals, and letters. In our case, we use black dots.
The black dots, as shown in the figure, indicate that for a given instant in time: ***when A_2 is positive, then a_1 is positive in the same instant of time.***



Note

Phase shift lead between phase voltages, V_{A2} and V_{a1}, Windings Placed on the Same Leg
The identification of polarity becomes essential when we operate the two transformers in parallel. Otherwise ***if terminals of unlike polarity connected to the same line, the two secondary windings would be short circuited on each other with a resulting excessive current flow.***

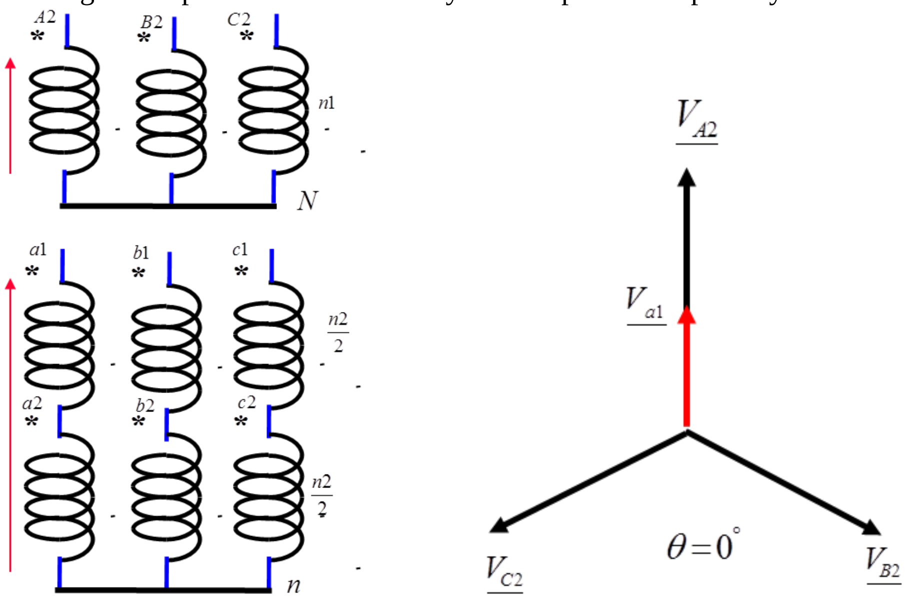
Because of the phase shift transformers cannot be connected in parallel unless they have proper phase sequence. (i.e. They are in phase with each other).

<u>Phase displacement</u>

Phase rotation is always clockwise (internationally adopted convention) and indicates multiples of 30 degree lag for secondary winding using the primary winding as the reference.
Thus 1 = 30°, 2 = 60°, 3 = 90°, 4 = 120°, 5 = 150°, 6 = 180°, 7 = 210°, 8 = 240°, 9 = 270°, 10 = 300°, 11 = 330° and 12 = 0° or 360°.
Transformers usually do not have the vector group shown on their nameplate and instead a vector diagram is given to show the relationship between the primary and other windings.
Application of Transformer according to Uses: Yy_0, Yd_{11}, Dy_{11}, Dd_0
A few examples of the physical connections and phasor diagrams are shown.

Star-Star (Yy_0)

Mainly used for large system tie-up transformer. In these transformers insulation cost is highly reduced. Neutral wire can permit mixed loading.
The angular displacement of secondary with respect to the primary are shown as clock position, 0°

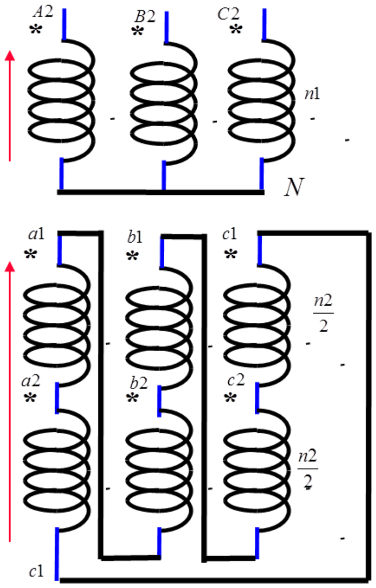


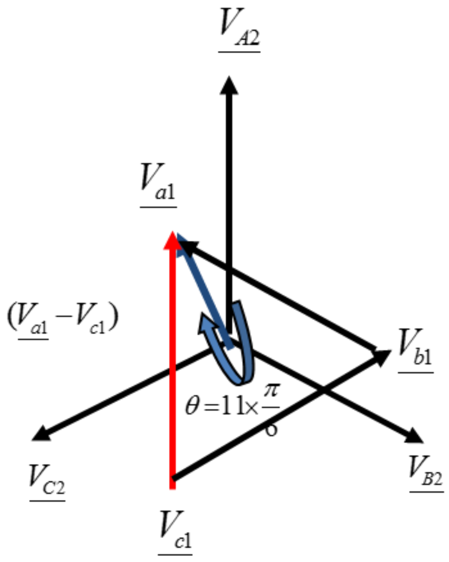
$$\frac{V_{a1}}{n_2} = \frac{V_{A2}}{n_1} \times e^{-j\theta} = \frac{V_{A2}}{n_1} \times e^{-j0^0}$$ As can be seen, there is a 0° phase relationship between V_{A2} and V_{a1}

$\frac{\underline{V}_{a1}}{\underline{V}_{A2}} = \frac{n_2}{n_1} \times e^{-j0^0} = me^{-j0^0}$ Therefor $m = \frac{n_2}{n_1}$ $\theta = 0^0$ and $H = \frac{0^0}{30^0} = 0$

Wye-delta (Yd_{11})

Mainly used for machine and main transformer in large power station and transmission substation.





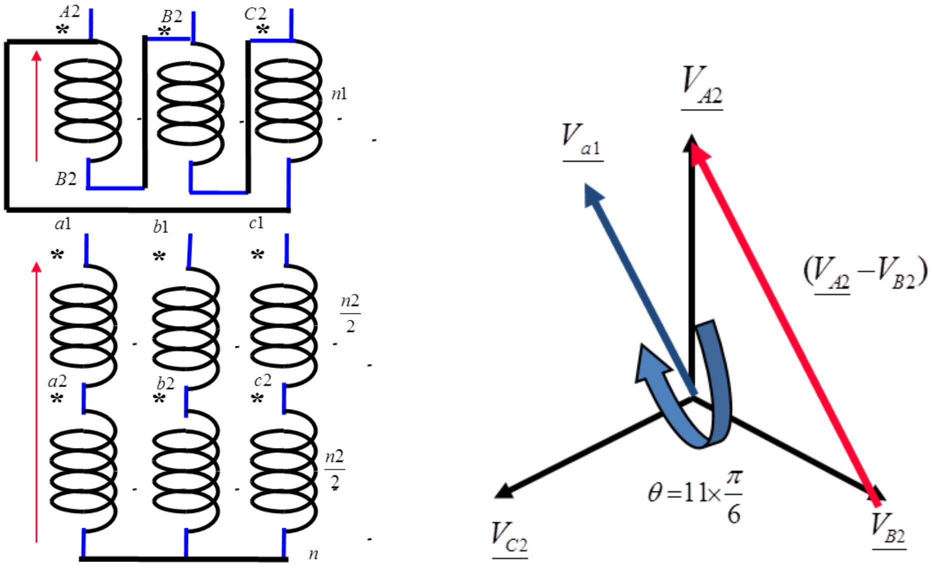
$\frac{\underline{V}_{a1} - \underline{V}_{c1}}{n_2} = \frac{\underline{V}_{A2}}{n_1} \times e^{-j\theta} = \frac{\underline{V}_{A2}}{n_1} \times e^{-j0^0}$ As can be seen, there is a 0° phase relationship between V_{A2} and $\underline{V}_{a1} - \underline{V}_{c1}$

$\frac{\sqrt{3}\underline{V}_{a1}}{\underline{V}_{A2}} = \frac{n_2}{n_1} \times e^{-j330^0} \rightarrow \frac{\underline{V}_{a1}}{\underline{V}_{A2}} = \frac{1}{\sqrt{3}} \frac{n_2}{n_1} \times e^{-j330^0} = me^{-j330^0}$ We can see, there is a 330° phase relationship between V_{A2} and $\underline{V}_{a1}$ therefor $m = \frac{1}{\sqrt{3}} \frac{n_2}{n_1}$ $\theta = 330^0$ and $H = \frac{330^0}{30^0} = 11$

Delta-star (Dy_{11})

Normally Dy_{11} vector group using at distribution system. Because generating transformer are Yd_1 for neutralizing the load angle between 11 and 1.

We can use Dy_{11} in distribution systems, when we are using generator transformer are Yd_{11}.

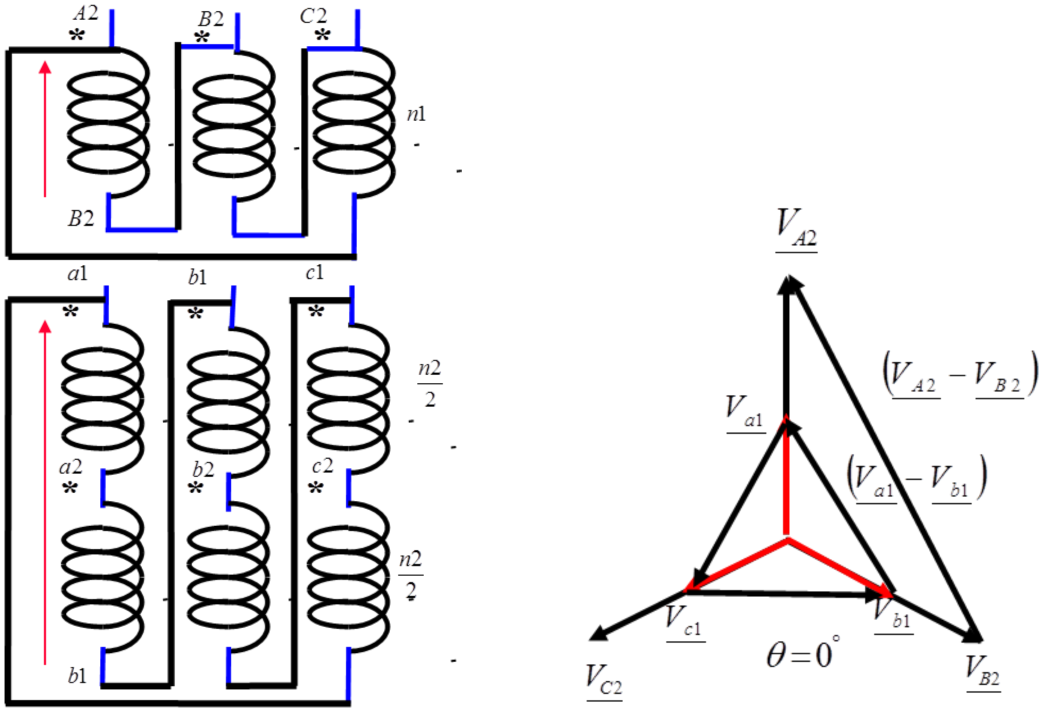


$\frac{V_{a1}}{n_2}=\frac{V_{A2}-V_{B2}}{n_1}\times e^{-j0}$ As can be seen, there is a 0° phase relationship between $(V_{A2}-V_{B2})$ and V_{a1}

$\frac{V_{a1}}{n_2}=\frac{\sqrt{3}V_{A2}}{n_1}\times e^{-j330^0} \rightarrow \frac{V_{a1}}{V_{A2}}=\sqrt{3}\frac{n_2}{n_1}e^{-j330^0}=me^{-j330^0}$ We can see, there is a 330° phase relationship between V_{A2} and V_{a1} therefor $m=\sqrt{3}\frac{n_2}{n_1}$ $\theta=330^0$ and $H=\frac{330^0}{30^0}=11$

Delta-delta (Dd_0)

Large unbalance of load can be met without difficulty. Delta permits a circulating path for triple harmonics thus attenuates the same.



$\frac{\underline{V}_{a1} - \underline{V}_{b1}}{n_2} = \frac{\underline{V}_{A2} - \underline{V}_{B2}}{n_1} \times e^{-j0}$ As can be seen, there is a 0° phase relationship between $\underline{V}_{A2} - \underline{V}_{B2}$ and $\underline{V}_{a1} - \underline{V}_{b1}$

$\frac{\underline{V}_{a1}}{\underline{V}_{A2}} = \frac{n_2}{n_1} \times e^{-j0^0} = me^{-j0^0}$ Therefor $m = \frac{n_2}{n_1}$ $\theta = 0^0$ and $H = \frac{0^0}{30^0} = 0$

Single-phase equivalent circuit of 3 phase transformers

Validity conditions:

- Identical transformers balanced source and load
- Only one phase variables are used, the other phases are similar.
- Equivalent Y-representation
- Line-to-neutral = phase voltage to secondary side.

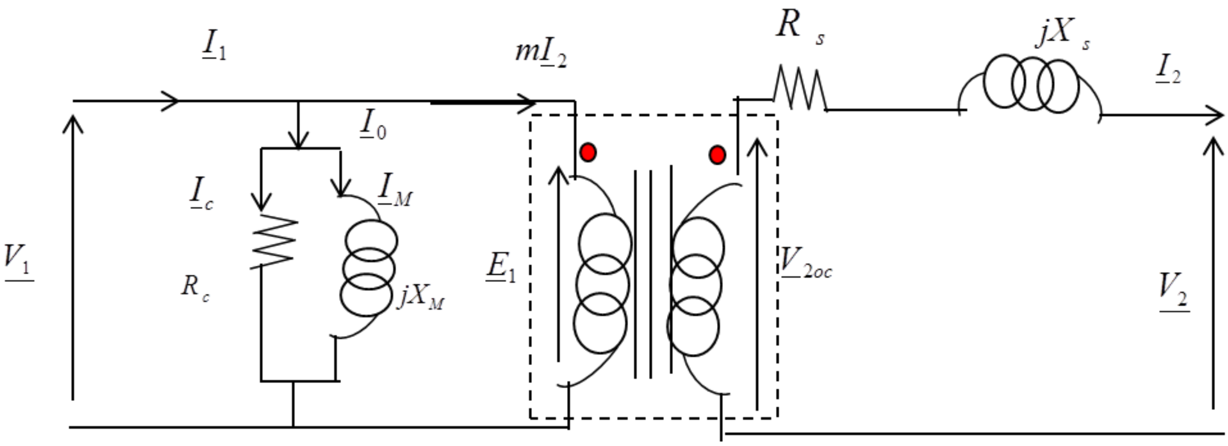


Figure 2 Single-phase equivalent circuit of 3 phase transformers

$m = \frac{U_{20}}{U_1}$; $R_c = \frac{U_1^2}{P_{oc}}$; $X_M = \frac{U_1^2}{Q_M}$ and $Q_M = \sqrt{S_{oc}^2 - P_{oc}^2}$; $S_{oc} = \sqrt{3}U_1 I_{1oc}$

$R_s = \frac{P_{sc}}{3 \times I_{2sc}^2}$; $X_s = \frac{Q_{sc}}{3 \times I_{2sc}^2}$ And $Q_{sc} = \sqrt{S_{1sc}^2 - P_{sc}^2}$; $S_{1sc} = \sqrt{3}U_{1sc} I_{1sc}$

Exercise 2.6. *A 150000 VA, 20000V/400 V, 50 Hz* transformer has been tested to determine its equivalent circuit. The results of the tests are shown below.

Open-circuit test	Short-circuit test
U_{1oc}=20000V, U_{2oc} = 420 V	U_{1SC} = 1000 V
I_{1OC} = 0.43 A	I_{2SC} = 206 A
P_{OC} = 2000 W = p_{core}	P_{SC} = 1500 W

(A) Find the single phase equivalent circuit of this transformer referred to the secondary side and gives the transformer parameters

(B) Assume that the load on the transformer is 126 kW at $I_2 = 196A$. Determine the transformer's efficiency at rated conditions.

Voltage Drop Calculations

Ohm's Law, expressed in phasor form for AC circuits, and gives the basic relationship for voltage drop and the load current.

$\Delta U = \sqrt{3}\left(R_s I_2 \cos\varphi_2 + X_s I_2 \sin\varphi_2\right)$

$\underline{\Delta U} = \sqrt{3}\left(\underline{Z_s} \times \underline{I_2}\right)$

$\underline{Z_s} = \left(R_s + jX_s\right)$

$\underline{\Delta U}$ is the voltage drop, in **volts**

$\underline{I}_2$ is the load current in **amperes**

$\underline{Z_s}$ is the conductor or equipment impedance, in **ohms**

Review 6

1. If the primary of a transformer has more turns than the secondary, it is a _________ transformer.
2. If the primary of a transformer has fewer turns than the secondary, it is a _________ transformer.
3. The secondary voltage of an iron-core transformer with 240 V on the primary, 40 A on the primary, and 20 A on the secondary is ________ V.
4. A single-phase transformer with a 480 V and a maximum load current of 20 A must have an apparent power rating of at least ________ kVA.
5. A wye-connected, three-phase transformer secondary, with 208 V line-to-line will have ________ V line-to neutral.

Keywords

Single-phase equivalent circuit of 3 phase transformer: Circuit equivalent monophasé d'un transformateur triphsé
Step up transformer: Transformateur élévateur
Step down transformer: Transformateur abaisseur
Input voltage: Tension d'entrée
Output voltage: Tension de sortie
Turns ratio: Rapport de transformation
Approximate transformer equivalent circuit: Schéma de Kapp d'un Transformateur
No load primary current: I_0 : Courant à vide au primaire
Magnetizing current: I_M : Courant de magnétisation
Current accounting for the core losses: I_c : Courant de pertes fer
Core or iron losses: R_c : Résistance de fer
Magnetizing reactance: X_M : Réactance de fer
Saturation: Saturation
Open circuit test: Essai à vide
Short circuit test: Essai en court-circuit
Transformer losses: Pertes du transformateur
Winding joule losses: Pertes joules
Hysteresis losses: Pertes par hystérisis
Eddy current losses: Pertes par courants de Foucauld
Transformer Ratings: Grandeurs nominales du transformateur

Basic Formulas

$$m = \frac{U_{20}}{U_1}$$

$$R_c = \frac{U_1^2}{P_{oc}}$$

$$X_M = \frac{U_1^2}{Q_M}$$

$$Q_M = \sqrt{S_{oc}^2 - P_{oo}^2}$$

$$S_{oc} = \sqrt{3} U_1 I_{1oc}$$

$$R_s = \frac{P_{sc}}{3 \times I_{2sc}^2}$$

$$X_s = \frac{Q_{sc}}{3 \times I_{2sc}^2}$$

$$Q_{sc} = \sqrt{S_{1sc}^2 - P_{sc}^2}$$

$$S_{1sc} = \sqrt{3} U_{1sc} I_{1sc}$$

$$P_1 = P_2 + P_{oc} + 3 \times R_s \times I_2^2$$

$$P_2 = \sqrt{3} U_2 I_2 \cos\varphi_2$$

$$\eta = \frac{P_2}{P_2 + P_{oc} + 3 \times R_s \times I_2^2}$$

References

This course uses numerous works and books on which I took photos or diagrams. I am thanking all the authors who contributed indirectly to the enrichment of my course.