Course
Electromechanical conversion of electric energy
Presented by :
Dr.Ing. Dhaker ABBES

Level : HEI4
Domain : ESEA
Referent teacher: Dr. Ing. Dhaker ABBES

Course objectives

- Understanding the operation of rotating machinery.
- Acquire the knowledge necessary for any engineer in Electrical Engineering, for modeling, simulation and control of conventional rotating machines in particular asynchronous and synchronous machines.
- Understanding the principle of automatic speed control of machines using electronic switches.

Outcomes

At the end of this course, the student should be able to :

- Make a judicious choice of equipment (machine, inverter, etc.) depending on the application to implement.
- Model, simulate, control and connect all correctly.
- Perform standardized tests on the machine and to determine any model.

Course Organization

Course Content :

- **Chapter 1 :** Electrical machines and rotating field
- **Chapter 2 :** Study of the synchronous machine in steady state
- **Chapter 3 :** Study of the induction machine in steady state
- **Chapter 4 :** Dynamic modeling of the asynchronous machine (study in any regime)
- **Chapter 5 :** Dynamic modeling of the synchronous machine (study in any regime)
- **Chapter 6 :** Power converters associated with electrical machines

Tutorials (practical labs) :

- **TP1 :** Performance (efficiency) of an asynchronous motor by the method of separate losses
- **TP2 :** Study of a triphase alternator
- **TP3 :** Study of a speed controller for an asynchronous machine: scalar control

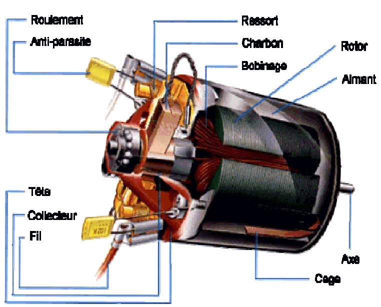
Mini project :

Modeling and dynamic simulation of a system with a three-phase AC machine

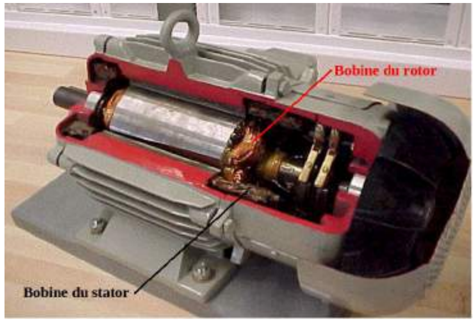
Course organization

Number of hours :	**Course :**	**10,5**	**TD :**	**10,5**	**TP :**	**9**	**Project :**	
Evaluation (in %) :	**Exam 1:**	**70**	**Exam 2 :**		**TP :**	**30**	**Project :**	
Module code :	**ESA021**							

Chapter 1:
Electrical machines and rotating field



Direct Current (DC) motor



Industrial synchronous Alternatif Current (AC) motor

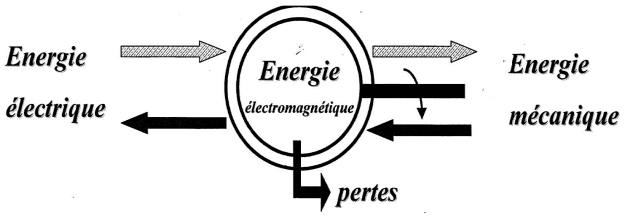
Three-phase asynchronous motor

1.1. Introduction

1.2. General informations on rotating electrical machines

1.2.1. Function:

An electric machine is an electromechanical device for converting **electrical energy** into **mechanical energy** or **work** (in the case of a motor) and the conversion of **mechanical energy** into **electric energy** (in the case of a generator). The operation of all electrical machinery is reversible.



Machine tournante

1.2.2. The three main types of electrical machines

■ **DC machines**

- Presence of excitation (winding traversed by a direct current or permanent magnet) in stator.
- Having a collector on the rotor which fixes the direction of the rotor field. The stator and rotor fields are fixed relative to the stator.

Notes

■ **AC machines :**

- **Synchronous machines**

➤ Presence of excitation (winding traversed by a direct current or permanent magnet) in the rotor.

➤ The rotor rotates at the same speed as the rotating field.

- **Asynchronous machines**

➤ No excitation.

➤ The rotor rotates at a different speed than the rotating field.

1.2.3. Description of a rotating electrical machine

■ **Mechanical point of view :**

➤ The stator is the fixed part of the machine, it is solid enough to not be moved by the action of the movable part.

➤ The rotor is the part of the machine which makes rotational movement. It is located inside the stator and is connected to the drive shaft.

➤ The space between the rotor and the stator is called air gap .

Electric machines have :
1- magnetic materials charged to drive and channel the magnetic flux,
2- conductive materials responsible for conducting and channeling electrical currents,
3- insulation materials, 4- a « container »: motor casing, 5- a cooling system.

1.3. General laws of electromechanical conversion

Rotating electrical machines convert mechanical energy into electrical energy and vice versa : they are generators (electric) or motors. This is an energy conversion with lower efficiency due to inevitable losses.

The following theoretical example used to materialize the various laws that regulate this conversion. A mobile conductor of ℓ length moves with $\boldsymbol{v}$ speed on two indefinite rails placed in a uniform and unchanging $\boldsymbol{B}$ induction, normal to conductors. A mechanical force is exerted $\boldsymbol{F_m}$ on the conductor and the circuit is powered by a generator of e.m.f $\boldsymbol{E_0}$ and internal resistance $\boldsymbol{R}$. We note $\boldsymbol{I}$ the current, with sign conventions shown in Figure.

Four laws determine the electromechanical system :

➤ **Faraday's Law** : If the speed of conductor is $\boldsymbol{v}$, it appears an e.m.f $E = B \times \ell \times \boldsymbol{v}$

➤ **Laplace's law** : if the current in the conductor is **I**, there are an electromagnetic force $\boldsymbol{F_e}$:

$$F_e = B \times \ell \times I$$

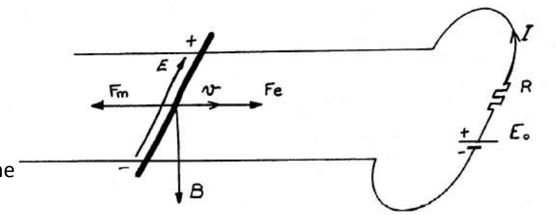


Figure 1 : Principle of electric power generator

➤ **Ohm's Law** : $\boldsymbol{E_0 = E + R.I}$

➤ **Dynamics law** : if the speed $\boldsymbol{v}$ is constant, it implies : $\boldsymbol{F_m = F_e}$

If the resistance is zero (no loss in the circuit), then :
- $\boldsymbol{v}$ speed as : $\boldsymbol{E_0 = E + R.I}$;
- **I** current as : $\boldsymbol{F_e = F_m}$.

The speeds are related to e.m.f and current to forces.

The operation will be motor if $\boldsymbol{F_e}$ and $\boldsymbol{v}$ are on the same direction (these are the conventions of the figure). E.m.f $\boldsymbol{E}$ then opposes the current.

If the speed is in the direction of the applied force $\boldsymbol{F_m}$, an electric generator is obtained; the electric force $\boldsymbol{F_e}$ then opposes F_m .

We can express the power at the conductor in the mechanical or electric form :

$$P = F_e v = \mathrm{B} \times \ell \times I \times \frac{E}{B\ell} = \mathrm{E} \times I$$

This is the electromagnetic power. Note that the energy conversion is completely reversible.

Notes

1.4. Production of electromotive forces

The rotation of conductors in rotating machines leads to the development of e.m.f. This section deals mainly with synchronous and DC machines. While it does include the same laws, asynchronous machines will be studied separately.

1.4.1. Machine structure

An electrical machine comprises:

A closed magnetic circuit : the induction lines cross the breech, the poles, air gaps, the rotor in a pattern shown in the diagram;

Excitation windings : that create the magnetic flux. These windings are arranged around the poles and are DC powered;

Rotor or armature : cylindrical, it contains in grooves (or notches) rectilinear conductors parallel to its axis of rotation. These drivers will then be interconnected.

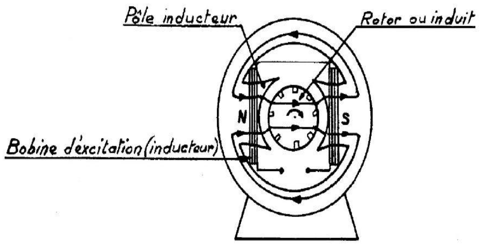


Figure 2 : Structure of an electrical machine

The rotor is driven at a constant rotation speed **N** (in revolutions per second) or ω (in radians per second) with $\boldsymbol{\omega = 2\pi N}$.

Electromotive forces are induced in the rotor conductors : they depend on the induction in the air gap and hence the instantaneous position of the considered conductor.

2.4.2.Magnetic flux and induced electromotive forces

We assume for the rest of the course that the flow is of sinusoidal distribution and given by : $\boldsymbol{\varphi = \emptyset \ \sin \omega t}$.

Regroup **n** conductors at the periphery of the armature, we obtain a larger e.m.f that is easy to collect by sliding contacts (brushes or coal) :

$$\boldsymbol{e = nN\emptyset \sin \theta_0 \cos \omega t}$$

This electromotive force is sinusoidal, with pulse $\boldsymbol{\omega = 2\pi N}$ and amplitude depending on the speed, flow and θ_0.

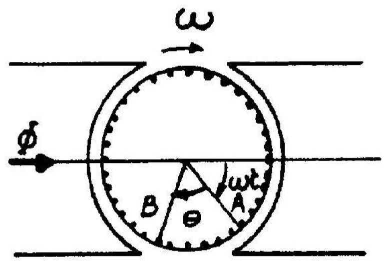


Figure 3 : e.m.f in a conductor

CONSEQUENCES :

1. If we put in series the n conductors, whether $\theta_0 = \pi$, $e = 0$, we can therefore close the armature circuit : no current will flow. The coil has no more ends : the rotational symmetry of the armature is perfect. This is evident if one considers that the induced e.m.f in the conductors cancel each other by symmetry relative (with respect) to the axis of rotation. The magnitude of the electromotive force is maximum for $\theta_0 = \frac{\pi}{2}$; $\frac{n}{2}$ conductors in series :

$$\boldsymbol{e = nN\emptyset \cos \omega t}$$

2. To collect the electromotive force, there are two solutions:
Synchronous machine : two conductive rings are connected to the winding in two fixed points ($+\theta_0$ et $-\theta_0$) as shown in Figure 4. Two fixed brushes rub on these rings and thus the AC voltage is collected :

$$\boldsymbol{e = nN\emptyset \sin \theta_0 \cos \omega t}$$

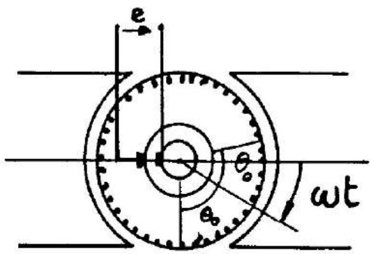


Figure 4 : Position of the brushes in a synchronous machine

Notes

DC machine : Two fixed blades are placed in contact with the conductors passing through the neutral line as shown in figure 5. So the induced e.m.f is collected at every moment in a half of armature ($\theta_0 = \frac{\pi}{2}$) and at the moment it is maximum ($\omega t = 0$).
We thus obtain a constant voltage value: :

$$E = nN\emptyset$$

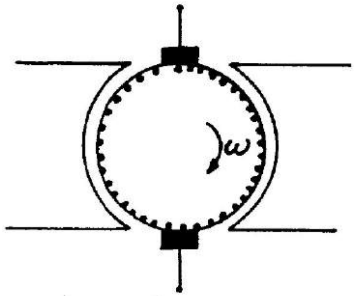


Figure 5 : Position of the brushes in a DC machine

Synchronous and DC machines does not therefore differ in their principles.

1.5. Rotating inductions

In this section, we will assume that the magnetic circuits are not saturated, allowing to speak either rotating magnetomotive forces or rotating inductions because they are proportional.

If there is saturation, we can speak only of magnetomotive rotating forces, from which are deducted, via the magnetization curves, flows and inductions..

1.5.1. Turning Inductor

The rotor has p pairs of poles (North-South pairs) whose windings supplied DC creates a succession of alternating north and south poles.

At time $t = 0$, induction at a point B of the gap is shown in Figure 6 : its period is $\frac{2\pi}{p}$ and its variation is assumed to be sinusoidal:

$$t = 0 \qquad b = B_m \cos p\theta$$

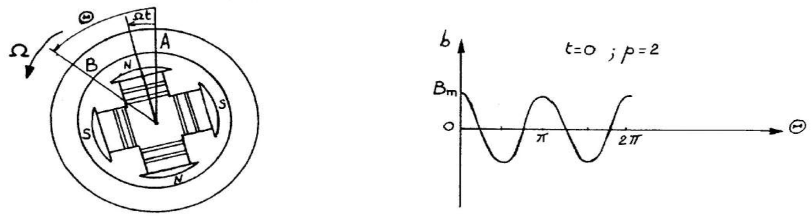


Figure 6 :Magnetic field created by a rotating inductor.

If the rotor rotates at the angular velocity Ω, the angle between point B and the north pole taken here as origin is $(\Theta - \Omega t)$ and induction in B is :

$$b = B_m \cos p\,(\Theta - \Omega t)$$

Taking $\omega = p\Omega$, we have : $b = B_m \cos\,(p\Theta - \omega t)$

Induction at a fixed point of the air gap varies sinusoidally at the pulsation ω , thus at the frequency $f = pN$ because $\Omega = 2\pi N$. Rotating inductions (or rotating magnetomotive forces) are engendered.

1.5.2. Fixed three-phase inductor

1.5.2.1. Three-phase inductor

Is fed by the three phases of a three phase network, three identical coils, each generating p pole pairs and staggered in space of 2π / 3p radians (120 ° / p).

Examples:

Phases are identified by the following symbols:

phase 1 : ○ ● ; phase 2 : △ ▲ ; phase 3 : □ ■

Bipolar inductor : one coil per phase thus generating a pair of poles per phase (p = 1). Ce These coils are shifted by 120 ° as shown in Figure 7 :

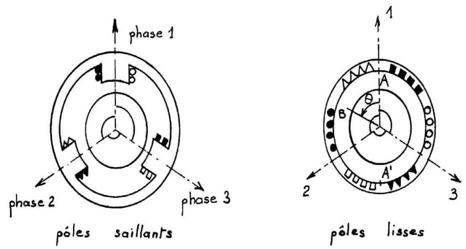


Figure 7 : Bipolar inductor with salient poles and with smooth poles

Notes

The inductor with smooth poles comprises coils placed in magnetic circuit slots; thus, by developing the lateral surface of the gap, we can give the schematic representation of figure 8 :

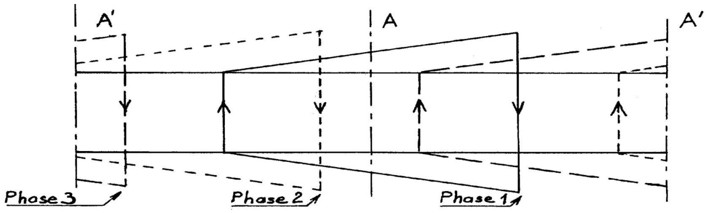


Figure 8 : expansion of the lateral surface of a smooth poles inductor

4-pole inductor : It comprises two coils per phase , therefore two pairs of poles per phase (p = 2). These coils are offset by 60 ° as shown in Figure 9 :

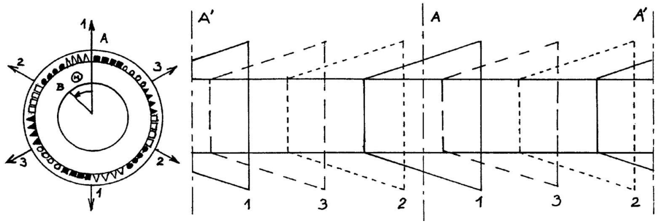


Figure 9 : expansion of the lateral surface of a 4-pole inductor

1.5.2.2. Ferraris Theorem

A three-phase fixed inductor having p pairs of poles per phase, supplied to the pulsation ω generates a rotating induction for the p pole pairs with a rotating speed (angular Ω_s) such that:

$$\boldsymbol{\Omega_s = \frac{\omega}{p} \quad N_s = \frac{f}{p}}$$

Examples :

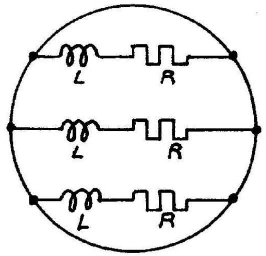
p \ fréquences	50 Hz	60 Hz	400 Hz	
1	3000	3600	24000	tr/min
2	1500	1800	12000	tr/min

1.5.2.3. Inductance of a three-phase winding

It can be attributed, in balanced functionning, an impedance specific to each stage: it is the cyclical or synchronous impedance. It implicitly takes into account the existing linkages with other phases.

A three-phase inductor therefore present, per phase, a resistance R and a self-inductance L. In practice, we have :

$$R \ll L\omega$$



1.6. Equivalent bipolar Machine

If in the above is posed:

$$\boldsymbol{\theta = p\Theta}$$

Inductions expressions are written :

$$\boldsymbol{b = A\cos(\omega t - \theta)}$$

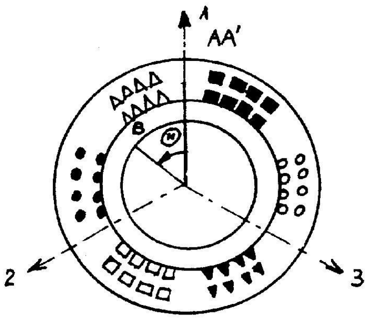
The induction would be generated by a bipolar plate, rotating with $\boldsymbol{\omega = p\Omega_s}$ speed and DC powered, or fixed and fed at ω pulsation with three-phase power.

Rotating inductions and Fresnel vectors then rotate at the same speed ω and we can therefore superimposed their diagrams.

For example, paragraph 1.5.2.1 **4-pole** inductor may be represented by an equivalent inductor pole having the same number of conductors:

$$\boldsymbol{\theta = p\Theta}$$
$$\boldsymbol{\omega = p\Omega_s}$$

θ and ω are the « electrical » angles and speeds. **Any machine can be studied based on this equivalent model.**



Notes

1.7. Microscopic explanation of functionning

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http://moodle.lfm.edu.mx/mod/resource/view.php?inpopup=true&id=1602

http://youtu.be/DcinrDuFlKs?t=11m50s

http://www.youtube.com/watch?v=LtJoJBUSe28

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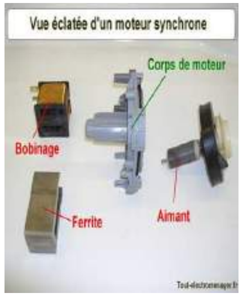
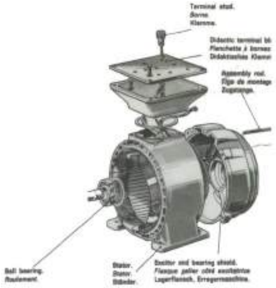
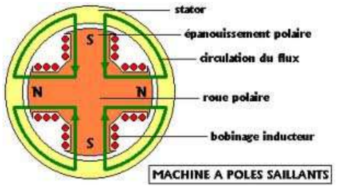
Chapter 2: Study of the synchronous machine in steady state





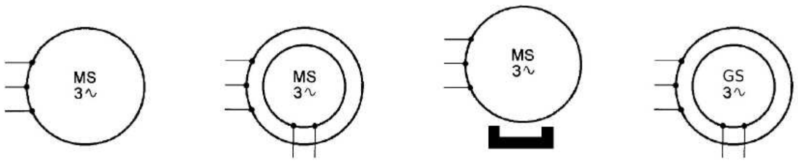


2.1. Introduction

The synchronous machine is most often used as a generator, then it is called alternator. The production plants are equipped with three-phase power generators. From bicycle alternator delivering only a few watt to nuclear power station providing 1.6 GW, operating principle and classical models are quite similar.

Like all rotating electrical machines, synchronous machine is reversible and can also function as a synchronous motor.

2.2. Symboles



(a) Symbole général d'un moteur synchrone. (b) Symbole d'un moteur synchrone triphasé à rotor bobiné. (c) Symbole d'un moteur synchrone triphasé à aimants. (d) Symbole d'un alternateur triphasé à rotor bobiné.

Figure 2.1: Symbols of the synchronous machine

2.3. Construction-Principe

- **The exciting winding :** it consists of a coil wound on the rotor and through which run the "excitation" direct current : *Ie*. This is what allows the creation of magnetic poles called "rotoric" and the introduction of a given flux in the magnetic circuit. This coil is sometimes replaced by permanent magnets especially in the field of small and middle powers.
- **The armature circuit :** it consists of the **3** three-phase coils, symmetrically constituted, practiced on the stator in a distributed manner, and through which passes the electric power of the machine.

Notes

2.3.1. Electromotive Force

$$e = nN\emptyset \sin\theta_0 \cos\omega t$$

This voltage is collected by sliding contacts (rings). Only the relative displacement of the armature relative to the inductor is important: thus we can get the same result by taking a fixed armature and a movable inductor. The realization will be easier : only the direct excitation current of the inductor cross sliding contacts. The armature may be more complex (three phase armature) and traversed by higher currents.

Note that the e.m.f is maximum when the poles are perpendicular to the axis of the formed coil.

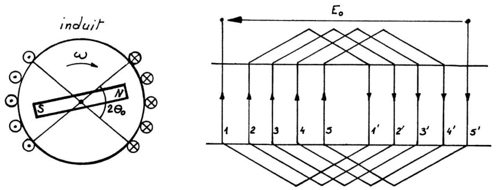


Figure 2.2 : Armature of a synchronous machine with 10 conductors

General case : alternator with 2p poles :
The inductor includes 2p poles and the armature has p times the previous winding:
for example p=2 ; n_1=20.
n_1: total number of conductors connected in series, different from n : total number of conductors disposed on the armature.

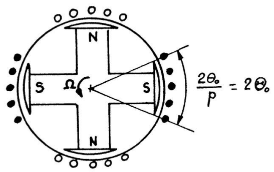


Figure 2.3: Winding example of a synchronous machine with two pairs of poles and 20 conductors in the armature.

This machine is equivalent to a bipolar machine for which : $\begin{cases}\omega = p\Omega_s \\ \theta = p\Theta\end{cases}$
and having the same number of conductors n_1. N_s becoming pN_s, electromotive force will be therefore written as :

$$e = \frac{\pi \sin\theta_0}{\theta_0} \times n_1 \times p \times N_s \times \emptyset \times \cos\omega t$$

with $\theta_0 = p\Theta_0$

Its Root Mean Square (R.M.S) value E is such that :

$$E = \left(\frac{\pi \sin\theta_0}{\sqrt{2}\,\theta_0}\right) \times n_1 \times p \times N_s \times \emptyset(I_e)$$

The factor in brackets, called Kapp coefficient takes particular account of conductors allocation to the stator periphery and thus the existing phase shifts between e.m.fs induced therein.
By introducing a coefficient k related to construction, we ultimately obtain:

$$\boldsymbol{E = kN_s\emptyset(I_e)}$$

with : $e = E\sqrt{2}\cos\omega t$

$$\boldsymbol{\omega = p\Omega} \text{ or } \boldsymbol{f = pN_s}$$

The latter formula, which connects the frequency, the number of poles and the rotational speed is identical to the rotating Ferraris inductions formula.

2.3.1.1. Caractéristique à vide

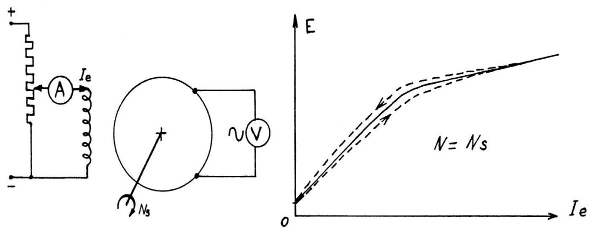


Figure 2.4: Vacuous (uncharged) characteristic of a synchronous machine

2.3.1.2. Alternateur triphasé

$$\begin{cases} e_1 = E\sqrt{2}\cos\omega t \\ e_2 = E\sqrt{2}\left(\cos\omega t - \frac{2\pi}{3}\right) \\ e_3 = E\sqrt{2}\left(\cos\omega t - \frac{4\pi}{3}\right)\end{cases}$$

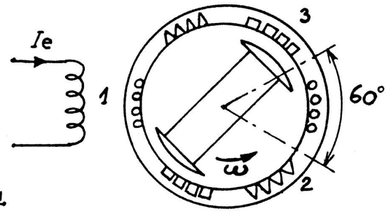


Figure 2.5: Three phase alternator : distribution of the three armature windings.

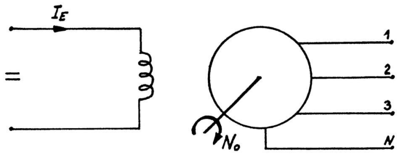


Figure 2.6: Three phase alternator : principal diagram.

Notes

2.3.1.3. Excitation of the synchronous machine

The inductor must be supplied with DC voltage. It can be used :

- A DC self excited generator.
- A small generator whose voltage is rectified. We can in this case not use brushes: the excitation alternator consists on a rotating armature; the voltage rectified by the diodes, excites the inductor of the main rotating generator as shown in Figure 2.7. The current i_e is supplied by an electronic controller responsible for maintaining the constant output voltage, the excitement alternator serves besides as power amplifier.
- Rectifiers, commanded or not, using an alternating voltage. In the case of a functioning in alternator, we can use the produced alternating voltage; we obtain an auto-excited assembly which obeys the same conditions of priming as generators with direct current (presence of residual flow in particular)..

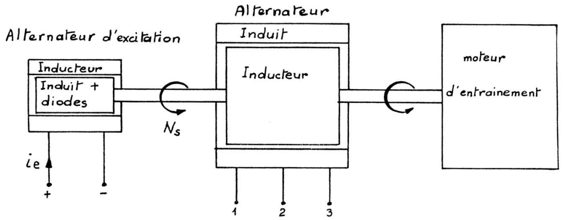


Figure 2.7: Principal diagram of a three-phase alternator without rings nor brushes.

2.3.2. Load operation

2.3.2.1. Equivalent circuit

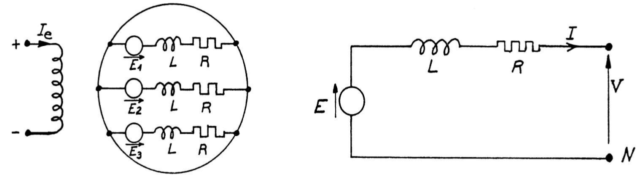


Figure 2.8 : Equivalent circuit of the synchronous machine

We will, by phase, assuming a current output I, with a phase $\varphi = (\widehat{\vec{V},\vec{I}})$:

$\underline{E}(I_e) = \underline{V} + R\underline{I} + \mathrm{j}L\omega\underline{I}$: This mesh equation gives the diagram of Behn-Eschenburg resembling that of Kapp for the transformer.

But here, because of the air gap, we have : $R \ll L\omega$.

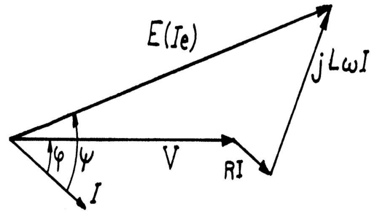


Figure 2.9 : Behn-Eschenburg diagram of the synchronous machine

Potier's method : When the alternator is saturated, we cannot any more, in any rigor, use the previous method. It is necessary to to compose the rotating magnétomotrice forces due to the inductor, in the rotor ξ_R and to armature, in the stator ξ_S , disorientated, them too, of $\frac{\pi}{2} + \Psi$.

We deduce the resultant magnetomotive force ξ_r , which, considering the magnetization characteristic, leads to the flow and to the resultant e.m.f E_r :

$$\underline{\xi_r} = \underline{\xi_R} + \underline{\xi_S}$$

By dividing this expression by the number of DC inductor turns, we reveal :

$I_e = \frac{\xi_R}{n}$ inductor current (DC);

$I_{er} = \frac{\xi_r}{n}$ inductor current resulting from the inductor and from the armature ;

$\alpha\, I = \frac{\xi_S}{n}$ equivalent DC in the armature.

$\underline{I_{er}} = \underline{I_e} + \alpha\underline{I}$ with fixed parameter α.

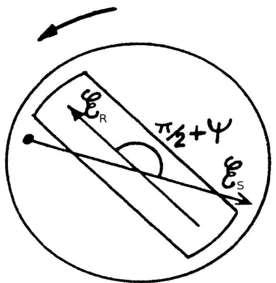


Figure 2.10 : Magnetomotive forces to establish the Potier's diagram of the synchronous machine

The e.m.f. in load E_r is given by the vacuous (uncharged) characteristic, for the value I_{er} of excitement current. The armature presents besides a constant flights inductance λ, favored by the air-gap.

Notes

Starting from V, I, φ, we construct E_r. We read I_{er} on the vacuous (uncharged) characteristic and we carry it with 90 ° beforehand E_r (already studied e.m.fs delay (shift)). We construct (αI) in phase with I and we get I_e.

We can complete the diagram by placing E, late of 90 ° *on* I_e, which reveals the angle Ψ.

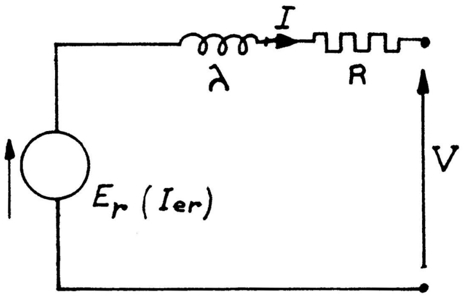


Figure 2.11 : Magnetomotive forces to establish the Potier's diagram of the synchronous machine.

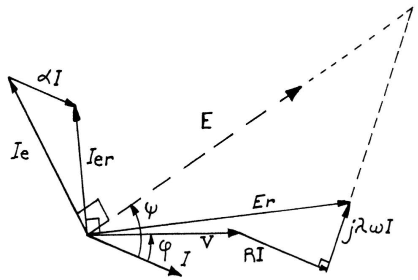


Figure 2.12 : Potier's diagram of the synchronous machine.

Potier's method, more rigorous when machines are saturated, led in longer calculations and more difficult to exploit.

2.3.2.2.Determination of equivalent circuit elements

The e.m.f $E(I_e)$ is known by the vacuous characteristic. We measure, in direct current, the resistance R of every winding. We could measure directly L ou $L\omega$ (impedance measure) but we prefer to use the following methods:

Essay in short circuit under reduced excitement We measure I_e and I_{cc}. We can draw the characteristic $I_{cc}(I_e)$.

The resultant flow being very low, the machine is not saturated and the characteristic in short circuit is rectilinear. We so obtain the internal phase impedance of the alternator :

$|R + jL\omega| = \frac{E(I_e)}{Icc(I_e)}$ since V=0 ; $\underline{E} = (R + jL\omega)\underline{I_{cc}}$

We deduce $L\omega$. By this calculation, we see that $L\omega$ is constant as long as there is no saturation (linear machine).

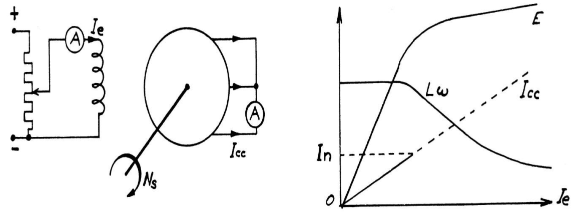


Figure 2.13 : Short circuit test of the synchronous machine.

Essay on inductance We make debit the machine on pure inductances.

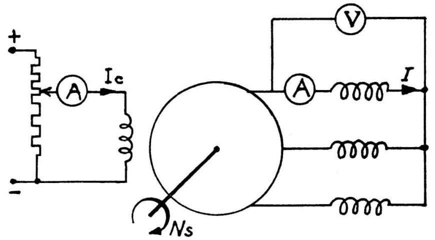


Figure 2.14 : Testing of synchronous machine: debit on pure inductances

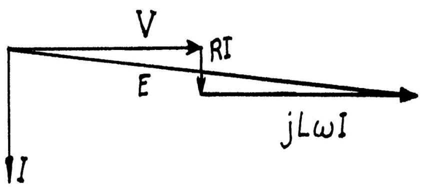


Figure 2.15 : Debit on pure inductances: Kapp diagram.

Given magnitude orders, we have : $L\omega I \approx E - V$.

Therefore, we measure $V(I_e)$ and by comparison with the vacuous curve, we deduce $(L\omega I)$ and $(L\omega)$. This method, more expensive because it requires bigger inductors, gives better results because $L\omega$ is measured under more normal flow situation than short-circuit test.

It means, in fact, that we implicitly take into account the non linearity by measuring $L\omega$ near normal conditions of functioning (close to the saturation). $L\omega$ is then a parameter, function of I_e, defined around an average point, much like the dynamic parameters in electronics (resistance of a diode).

Notes

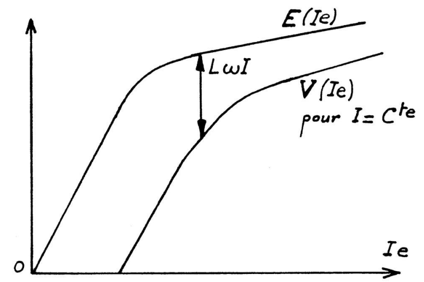


Figure 2.16 : Testing of synchronous machine: debit on pure inductances.

2.3.2.3. Electromagnetic torque

If the machine supplies the three-phase currents I, under the simple tensions V, with a phase shift φ,
Si la machine fournit les courants triphasés I, sous les tensions simples V, avec un déphasage φ, we have:

Supplied electrical power : $\boldsymbol{P = 3VIcos\varphi}$.

Joule losses in the armature : $\boldsymbol{P_j = 3RI^2}$.

The sum of these two powers comes from the drive motor which also provides the mechanical losses p_m : $\boldsymbol{P_{méca\,fournie} = P + P_J + p_m}$.

The electromagnetic power P_{em}, corresponding to the electromagnetic torque C_{em} is written as :

$$\boldsymbol{P_{em} = P + P_j = P_{méca\,fournie} - p_m = C_{em} \times \Omega_s}$$

This gives a precise expression of the torque, but rarely used for a general reasoning. If losses are neglected (implying an efficiency equal to one), we have :

$$\boldsymbol{P_{em} = C_{em} \times \Omega \approx 3VI\cos\varphi}$$

In this case, the diagram Behn-Eschenburg is simplified (RI is negligible) as shown in figure 2.17 :

$HM = L\omega I.\cos\varphi = E\sin\theta$

From where : $P_{em} \approx \frac{3V}{L\omega} \times \overline{HM}$

And torque can be written::

$$C_{em} = \frac{3V}{L\omega\Omega_s} \times \overline{HM}$$

Or, finally, in view of $\Omega_s = \frac{\omega}{p}$:

$$\boldsymbol{C_{em} = \frac{P_{em}}{\Omega_s} \approx (\frac{3p}{L\omega^2}) \times VE\sin\theta}$$

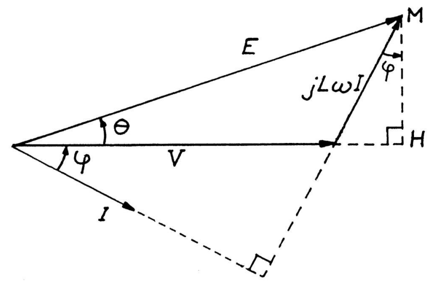


Figure 2.17 : Simplified Behn-Eschenburg diagram for calculating the electromagnetic torque of the synchronous machine.

2.4. Autonomous alternator and coupling to the grid

4.3.1. Autonomous alternator

The synchronous machine, driven at N_s speed by a motor (usually thermal) feeds a three-phase load with a power factor $\cos\varphi$. If we want to insure an imposed tension U , for a debit I and a phase shift φ, we construct the diagram of Behn-Eschenburg which gives the necessary e.m.f E and as a consequence the excitement current I_e.

Conversely, if I_e, so E, is fixed, we can calculate, by this diagram, the voltage obtained for given debit I and $\cos\varphi$:

$$\underline{E} = \underline{V} + R\underline{I} + jL\omega\underline{I}$$

We construct $R\underline{I}$ and $jL\omega\underline{I}$. φ is known and therefore the direction of V : $|\underline{E}|$ is known. We deduce the graphic solution of Figure 2.19.

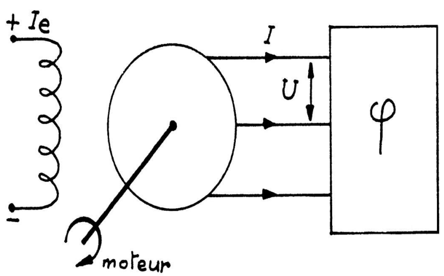


Figure 2.18 : Autonomous alternator.

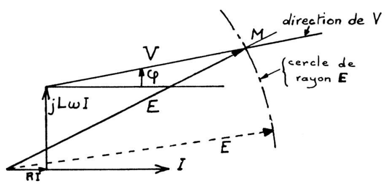


Figure 2.19 : Autonomous alternator : Behn-Eschenburg diagram for calculating E.

Notes

These two types of studies lead to the following curves::

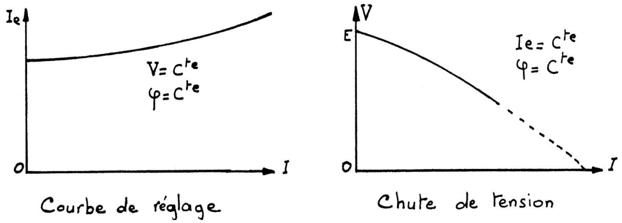


Figure 2.20 : Adjustment curve and voltage drop of an autonomous alternator synchronous machine.

4.3.2. Synchronous machine connected to the grid

A synchronous machine is coupled to a three-phase network to either :

- provide additional power to the network: the synchronous machine behaves in alternator;
- produce mechanical power: the synchronous machine then behaves as a synchronous motor.

These functionings are perfectly reversible and require both the same operations of coupling.

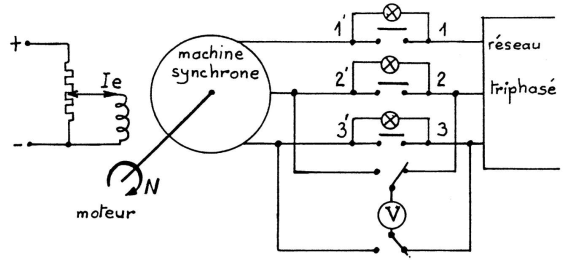
Coupling :

We cannot connect directly the stator of a synchronous machine on a network : currents would be too intense (limited only by the internal impedance $R + jL\omega$) and if the rotor speed differs from N_S, it may be no torque: the inductions B_R and B_S rotating at different speeds, The couple would be oscillatory and with zero mean value.

To achieve an optimal coupling of a synchronous machine, it is necessary to :

- train it at a speed *N* close to N_s by an auxiliary motor ;
- excite it to produce e.m.f. equal to network tensions;
- couple when e.m.f. and corresponding network tensions are in phase.

There is no current flowing in the armature. The practical assembly is the one of figure 2.21.



Figue 2.21 : Coupling of a synchronous machine on a network: a practical mounting

To verify :

- Speed : $N = N_s$;
- Phases sequences (1,2,3 for the grid and 1', 2', 3' for the synchronous machine) ;
- Phases equality : $arg\underline{E} = \arg\underline{V}$

We use coupling lamps that are placed in parallel on the circuit coupling breaker. These lamps are supplied with voltages : $\underline{U'} = \underline{V} - \underline{E}$ as shown in Figure 2.22.

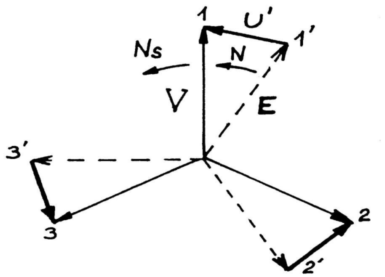


Figure 2.22 : Network and the synchronous machine tensions during the coupling.

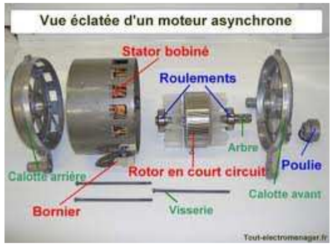
By acting on I_e , we settle the equality $|E| = |V|$ using the voltmeter.
As *N* is different from N_s , the three tensions U' vary simultaneously and the three lights will turn on and off together.

N speed is adjusted by turning the controller of the drive motor (+ speed - fast) so that the brightness of the lamps varies slowly and we couple with lamps extinction. We have then :

$$\underline{E}=\underline{V}$$

Notes

Chapter 3:
Study of the induction machine in steady state



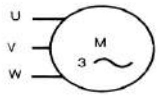
3.1. Introduction

Asynchronous machines are widely used (we estimate that 80 % of planet engines are asynchronous machines) because their cost is lower than other machines and are more robust.

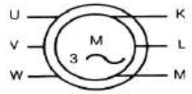
Like other machines, induction motor is reversible and numerous asynchronous generators of less than about 5 MW provide active energy surplus both on terrestrial networks and on ships board.

The power range covered by asynchronous machines is extensive : from some 5 watts for asynchronous AC motors with rings of phase shifts, to 36.8 MW squirrel cage motors of future British aircraft carriers of the "HMS Queen Elizabeth" class, to 24 MW squirrel cage induction motors propelling the series of ships 'Norwegian Epic'.

3.2. Symboles



(a) Symbole du moteur asynchrone à cage d'écureuil.



(b) Symbole du moteur asynchrone à rotor bobiné.

Figure 3.1 : Symbols of the induction motor

a) Squirrel cage asynchronous motor
b) Wound rotor induction motor

3.3. Structure- Operating Principles

3.3.1. Structure

- a three-phase stator having p pairs of poles per phase, identical with that of a synchronous machine;
- A rotor consisting of conductors placed in a closed circuit. We meet two types of rotor :

➡ Wound rotor: the winding, similar to that of the stator, contains p pairs of poles by phase ; the three pairs are connected with three rings which allow to insert a rheostat into the rotorique circuit. This motor is also known as motor with rings.

➡ Squirrel-cage : The rotor is made of copper or aluminum bars connected at both ends by two conductive rings. This model (like squirrel cage) inexpensive and very robust is the most common.

To avoid weakening the stator magnetic field due to a too large reluctance, the rotor is filled with discs of thin steel sheets (2 to 3 tenths of a millimeter) and electrically isolated by chemical surface treatment (phosphatation).

For instance, iron is the less reluctant material .

Notes

3.3.2. Principles of Operation

The stator, powered by a network of a frequency f, creates a rotating induction B_s of speed N_s such as $N_s = \frac{f}{p}$.

Let us suppose the rotor immobile : it is swept by this induction and electromotive forces are generated in the conductors (Faraday's law $e = \frac{d\varphi}{dt}$).

As the rotor circuits are closed, the rotor currents originate. It appears electromotive forces due to the action of stator induction on rotor currents. According to Lenz's law, these forces tend to drive the rotor in the direction of rotating inductions. There is a starting torque, the rotor starts turning if the couple is sufficient.

So that there is torque, it is thus necessary that :

- rotor circuits are closed, otherwise the rotor currents are zero;
- rotor speed N is different from induction speed N_s. If $N = N_s$, conductors rotate at the speed of the stator induction, no e.m.f is induced, and therefore no current flows in the rotor: it might be no torque.

We thus obtain a result very different from that of the synchronous machine for which there was torque at synchronism. For asynchronous machine:

- if $N < N_s$ motor torque;
- if $N = N_s$ zero torque;
- if $N > N_s$ braking torque.

3.3.3. Reminder of the main formulas

Slip : $g = \frac{N_s - N}{N_s} = \frac{\Omega_s - \Omega}{\Omega_s}$

Note that :
N=0 ; g=1 start-up
$N = N_s$; g=0 synchronism
$0 < N < N_s$; $0 < g < 1$ motor
$N > N_s$; $g < 0$ generator

Rotor frequency : f_R=g f

Power balance :

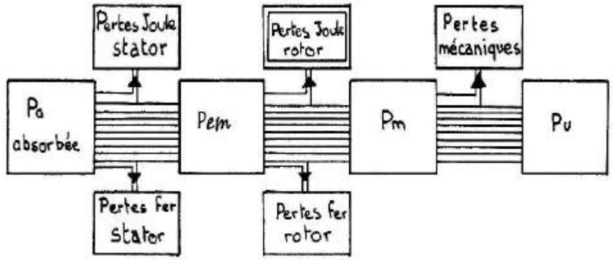


Figure 3.2 : Diagram of power balance of an asynchronous machine

- Absorbed power : $\boldsymbol{P_a = 3V_1 I_1 \cos\varphi_1}$
- Joule losses of the stator : if R_1 is the resistance of a stator phase, then $\boldsymbol{P_{js} = 3R_1 I^2{}_1}$.
- Stator iron losses: as with the transformer, they will be related to the square of the voltage : $\boldsymbol{P_f}$.
- Electromagnetic power P_{em}, It is the power transmitted by the stator to the rotor by the rotating inductions at the speed N_s : $\boldsymbol{Pem = C_{em}\frac{2\pi N_s}{60}}$.
- Joule losses of the rotor : if R_2 is the resistance of a rotor phase and I_2 the rotor current, we have : $\boldsymbol{P_{JR} = 3R_2 I^2{}_2}$.
- Rotor iron losses : they are low in normal functioning because the rotor frequency is small. We shall neglect them in practice in front of joule losses in rotor conductors.
- The mechanical power is supplied by the rotor with the speed N : $\boldsymbol{P_m = C_m\frac{2\pi N}{60} = C_m\,\Omega}$.
- The mechanical losses correspond to a friction torque $\boldsymbol{C_f}$.
- The useful power, delivered to motor output shaft, is written by introducing the useful torque : $\boldsymbol{P_u = C_u\frac{2\pi N}{60} = C_u\Omega}$.

We have obviously : $\boldsymbol{C_u = C_m - C_f}$. The dynamic balance of the rotor implies the equality of torques C_{em} and C_m. It results from it a remarkable property of the motor :

$$\boldsymbol{P_{em} = C_{em}\frac{2\pi N_s}{60} = P_m + P_{JR} = C_{em}\frac{2\pi N}{60} + P_{JR}}$$

Notes

$$P_{JR} = C_{em}\frac{2\pi(N_s - N)}{60} = C_{em}\frac{2\pi N_s}{60} g N_s = g P_{em}$$

$$P_{JR} = g P_{em} = g C_{em}\Omega_s$$

Efficiency :

$$\eta = \frac{P_u}{P_a} = \frac{P_u}{P_m} \times \frac{P_m}{P_{em}} \times \frac{P_{em}}{P_a} < \frac{P_m}{P_{em}}$$

Or : $\frac{P_m}{P_{em}} = \frac{N}{N_s} = 1 - g$ Donc $\eta < \frac{N}{N_s}$

Equivalent circuit :

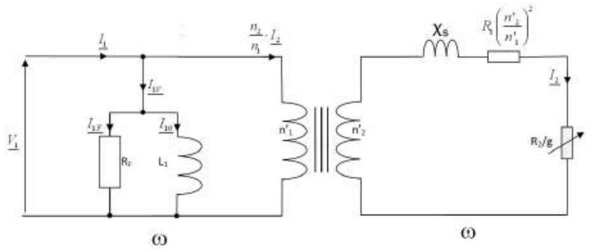


Figure 3.3 : Equivalent diagram of an asynchronous machine, stator and rotor are in the same angular frequency ω.

Torque and current at low slip :

It is assumed that : $\frac{R_2}{g} \gg \chi_s$ ou $g \ll \frac{R_2}{\chi_s\,\omega}$ $\left(R_1\left(\frac{n'_2}{n'_1}\right)^2\right)$ being a priori neglected.

We obtain : $\underline{I_2} = \frac{n'_2}{n'_1}\underline{V_1}\frac{g}{R_2}$ **soit :** $\underline{I_1} = \underline{I_{1V}} + (\frac{n'_2}{n'_1})^2\underline{V_1}\frac{g}{R_2}$

$$P_{em} = 3\frac{R_2}{g}I^2{}_2 = 3\frac{R_2}{g}(\frac{n'_2}{n'_1})^2\frac{g^2}{R^2{}_2}\underline{V^2{}_1} = C_{em}\Omega_s$$

Whether : $C_{em} = \frac{3}{\Omega_s}(\frac{n'_2}{n'_1})^2 V^2{}_1\frac{g}{R_2}$ **ou** $C_{em} = kV_1{}^2\frac{g}{R_2}$

For a given network (V_1 and Ω_s constants), the torque is proportional to the slip if R_2 is constant.

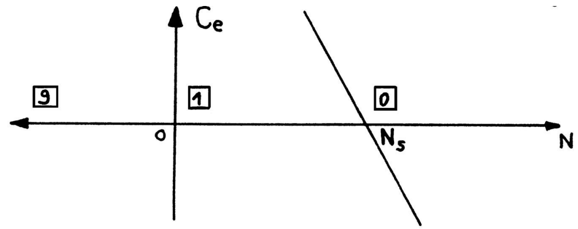


Figure 3.4 : Mechanical characteristic of an asynchronous machine for low slip

3.4. Absorbed current – circle diagram

3.4.1. Absorbed current

We have :

$$\underline{I_1} = \underline{I_{1V}} + (\frac{n'_2}{n'_1})^2\underline{V_1}\frac{1}{\frac{R_2}{g} + j\omega X_s}$$

Or : $\underline{I_1} = \underline{I_{1V}} + \underline{I'_1}$

When the speed varies, only $\underline{I'_1}$ component varies.

Consider : $\underline{I'_{1\infty}}$, the value of $\underline{I'_1}$ when $g = \infty$

$$\underline{I'_{1\infty}} = (\frac{n'_2}{n'_1})^2\frac{V_1}{j\omega X_s}$$

This current, limited by leakage inductance, is out of phase of 90° from V_1 and is constant (independent of g).

$$\underline{I'_1} = \underline{I'_{1\infty}}.\frac{1}{\frac{R_2}{jg\omega X_s} + 1} = \underline{I'_{1\infty}}.\frac{1}{1 - j\frac{R_2}{g\omega X_s}}$$

Notes

So :

$$\underline{I'_{1\infty}} = \underline{I'_1} - j\frac{R_2}{g\omega X_s} . \underline{I'_1}$$

This amount corresponds to Fresnel diagram according to Figure 3.5.

As I'_∞ is constant, the point M, extremity of I'_1, describes a circle of diameter $\boldsymbol{AB} = \boldsymbol{I'_\infty}$. The tangent of the angle α is directly proportional to the slip if R_2 is constant:

$$\tan\alpha = \frac{\omega X_s}{R_2} . g$$

If g = 0, M is A ; $g = \infty$, M is B.

Considering the absorbed current without load, I_{1V}, we will have for I_1 the final diagram in figure 3.6.

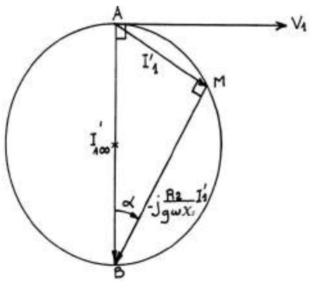


Figure 3.5 : Fresnel diagram of an asynchronous machine: the circle diagram

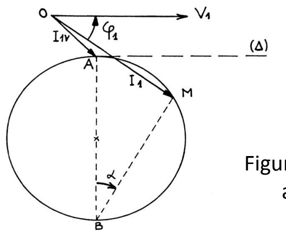


Figure 3.6 : Circle diagram of an asynchronous machine.

3.4.2. Experimental determination

The center being on a normal to V_1 crossing through A, we determine two points of the circle.

Vacuous test without load for $N = N_s$ $(g = 0)$

In practice, we let the motor run without load, as frictions are low, the slip is practically nul. We measure I_{1V} , power P_{1V} is equal to stator iron losses:

$$P_{1V} = 3V_1 I_1 \cos\varphi_{1V}$$

$$P_{1V} = 3V_1 I_{1F}$$

We can thus draw the vector $\overrightarrow{I_{1V}}$.

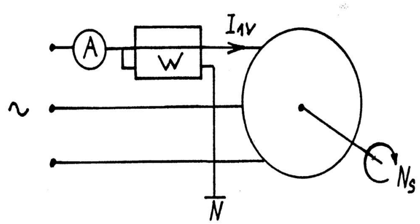


Figure 3.7 : Vacuous test for determining the circle-diagram.

Locked rotor test for N=0 (g=1)

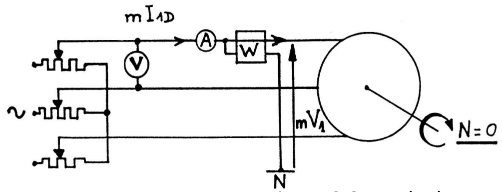


Figure 3.8 : Locked rotor test for determining the circle-diagram.

The ammeter measures mI_{1D}, the voltmeter mU. We deduce I_{1D}, while the wattmeter can calculate the dephasing. We carry then the vector $\overrightarrow{I_{1D}}$ and we construct the cercle. A and D are known, the center C of the circle is on the normal to V_1 and the mediator of AD as shown in Figure 3.9.

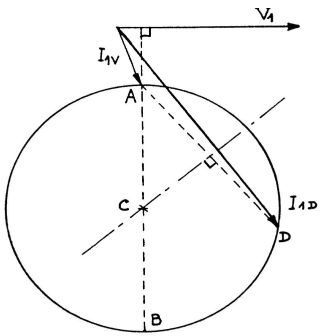


Figure 3.9 : Construction of the circle following the two trials : without load and with locked rotor.

Notes

3.4.3. Slip scale - Powers

Slip scale

(G) is graded, we will read the slip for a point M at the intersection of BM and (G).
We can so follow the evolution of I_1 and of φ_1 according to g, thus to the speed.

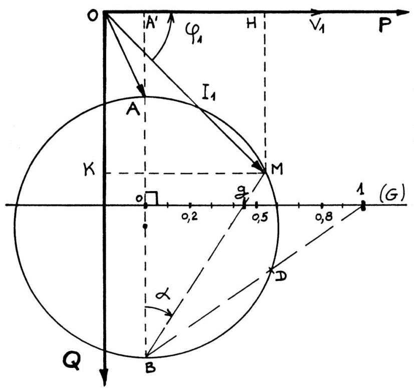


Figure 3.10 : Slip scale on the circle diagram.

Powers

As in the case of the simplified diagram, can be attached to this diagram scales for powers (see figure 3.10). Point M is projected on OP and OQ :

Absorbed active power : $P_a = 3V_1 I_1 \cos\varphi_1 = 3V_1\overline{OH}$

Absorbed reactive power : $Q_a = 3V_1 I_1 \sin\varphi_1 = 3V_1\overline{OK}$

When subtracting the stator iron losses, the electromagnetic power P_{em} is revealed on the axis OP:

$$p_F = 3V_1 I_{1F} = 3V_1\overline{OA'} \qquad\qquad P_{em} = P_a - p_F$$

$$P_{em} = 3V_1\overline{A'H}$$

$\overline{A'H}$, on the same scale, measures P_{em} and thus the torque. It can therefore be inferred from the diagram the torque-speed curve $C_{em} = f(g)$ ou $C_{em} = f(N)$.

Finally, joining A to D, can be separated rotor Joule losses and mechanical power P_m.

If x is the intersection of AD and mM, we have :

$$\frac{mx}{mM} = \frac{\tan\alpha}{\tan\alpha_D} = \frac{g}{1} = g$$

since $\widehat{mMA} = \alpha$ and $\widehat{mxA} = \alpha_D$.

As $\overline{mM}$ represents power P_{em}, $\overline{mx}$ represents the rotor Joule losses; $\overline{xM}$ corresponds then to mechanical power P_m :

$$P_{JR} = 3V_1\overline{mx} \qquad \text{and} \qquad P_m = 3V_1\overline{xM}$$

In summary, as shown in Figure 3.11, from the graph, can be obtained for a point M:

- Slip (scale G) ;
- Current I_1;
- Phase shift φ_1;
- Absorbed power P_a ;
- Electromagnetic power P_{em} and thus the torque;
- Rotor Joule losses;
- Mechanical power P_m.

We find directly the balance assessment of powers (with the exception of Joule stator losses which were neglected and mechanical losses which intervene after).

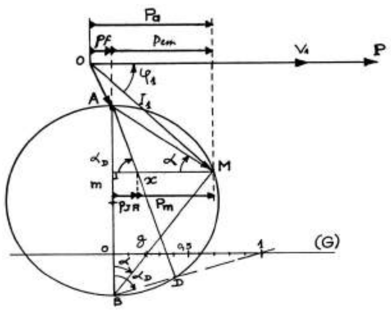
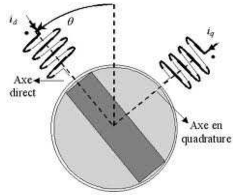
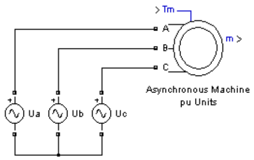


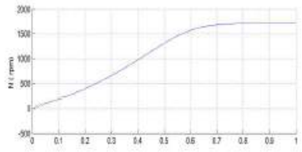
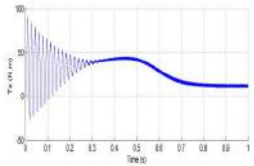
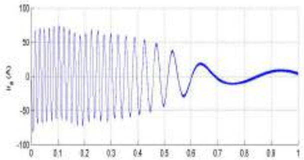
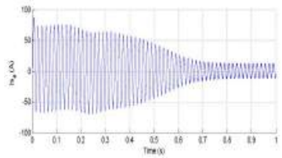
Figure 3.11 : All information from the circle diagram.

Notes

Chapter 4:
Dynamic modeling of the asynchronous machine
(study in any regime)





4.1. Introduction

The asynchronous machine model presented above is a model in “permanent regime" where the machine is supposed to work in steady state at constant speed as supplied with a system of three-phase voltages with constants RMS. The sizes are then sinusoidal and the approach in the complex space is valid (vectors of Fresnel). The equations of the machine, set in "steady state" used to calculate the torque and forecast operating points with a simple method, described $\frac{V}{f} = cte$ or also called scalar control. This allows to vary the speed of the machine over a wide range.

Another control method, based on the equations of "transient regime" (or "dynamic mode") of the machine is called “vector control”. It provides a response with faster dynamics, it also allows better precision in controlling the torque and in particular to obtain a torque at zero speed. However, this command is more difficult to implement since it requires more computing power to the microcontroller or DSP controller.

The name of vector control comes from that the final relations are vectorial (amplitude and phase) unlike the scalar control (amplitude only).

There are vectorial commands for asynchronous motors but also for synchronous engines.

4.2. Asynchronous machine model in transitory regime

4.2.1. Working hypotheses

- The winding is distributed so as to give a sinusoidal magnetomotive force (m.m.f) when powered by sinusoidal currents.
- We work in unsaturated state.
- We neglect the hysteresis phenomenon, Foucault currents and skin effect.
- Finally, homopolar regime is null because the neutral is not connected.

These choices mean also that :

- Flows are additive;
- Self inductances are constant;
- the variation of mutual inductances between stator and rotor windings as a function of electrical angle of their magnetic axis is sinusoidal.

Notes

4.2.2. Equations of asynchronous machine in any regime

4.2.2.1 Three-phase two-phase transformation

A rotating magnetic field can be obtained by a three-phase winding (three coils whose axes are angularly offset from $\frac{2\pi}{3}$ and supplied by tensions Moved in the time of 2/3 period) or by a two-phase winding (two coils offset by an angle of $\frac{\pi}{2}$ and fed by tensions shifted by a quarter period).

There are mainly two transformations :

- Clarke transformation ;
- Concordia transformation.

Clarke's transformation preserves sizes amplitude but not power nor torque, whereas that of Concordia keeps power but not amplitudes.

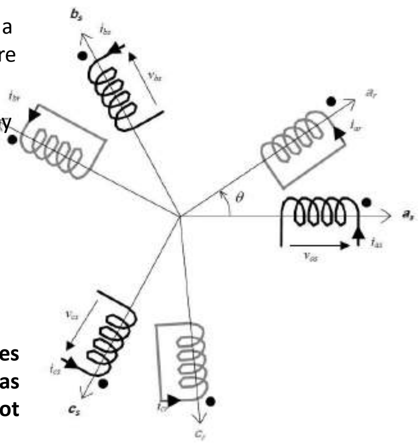


Figure 4.1 : Representation of stator and rotor windings of an asynchronous wound-rotor machine.

4.2.2.2 Clarke transformation

$$[Cl] = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

This will give for currents :

$$\begin{bmatrix} i_\alpha \\ i\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix}$$

Soit : $[i_{\alpha\beta}] = [Cl][i_{abc}]$

The coefficient $\frac{2}{3}$ is arbitrary, but it is adopted because it keeps current amplitude. A balanced three-phase system of sinusoidal currents of amplitude I_M produces a current vector of amplitude I_M.

Inverse transformation is thus :

$$\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

Means : $[i_{abc}] = [Cl]^{-1}[i_{\alpha\beta}]$

4.2.2.3 Concordia transformation

The definition for current is :

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_o \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} \sqrt{2} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{\sqrt{3}}{\sqrt{2}} & -\frac{\sqrt{3}}{\sqrt{2}} \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

or :

$$[i_{\alpha\beta o}] = [Co]\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Inverse transformation is :

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} \sqrt{2} & 0 & 1 \\ -\frac{1}{\sqrt{2}} & \frac{\sqrt{3}}{\sqrt{2}} & 1 \\ -\frac{1}{\sqrt{2}} & -\frac{\sqrt{3}}{\sqrt{2}} & 1 \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \\ i_o \end{bmatrix}$$

or :

$$[i_{abc}] = [Co]^{-1}[i_{\alpha\beta o}] = [Co]_t[i_{\alpha\beta o}]$$

Notes

4.2.2.6 Park transformation

Park transformation consists of a three phase-two phase transformation followed by a rotation. It allows to pass from abc reference to $\alpha\beta$ reference then dq reference. $\alpha\beta$ reference is always fixed compared with abc reference, while dq is mobile. As shown in Figure 4.2, it forms with the fixed reference αβ an angle which is called the angle of Park transformation or Park angle.

3x3 rotation matrix will be written :

$$R(\theta)=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

We will therefore have:

$$\begin{bmatrix}i_d\\ i_q\\ i_o\end{bmatrix}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}i_\alpha\\ i_\beta\\ i_o\end{bmatrix}$$

i_d direct axis and i_q the axis in quadrature « advance ». i_o represents homopolar component perpendicular to the plane of figure 4.2. We note that $[R(-\theta)]=[R(\theta)]^{-1}$.

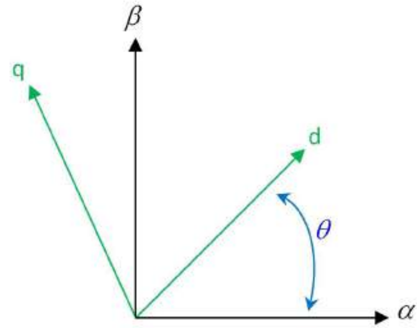


Figure 4.2 : Rotation of α-β to d-q system.

Concordia transformation preserves the instantaneous power. We shall use this transformation to realize Park transformation which keeps the power.

$$[P(\theta)]=[R(\theta)][C_o]$$

For currents this would be :

$$\begin{bmatrix}i_d\\ i_q\\ i_o\end{bmatrix}=\sqrt{\frac{2}{3}}\begin{bmatrix}\cos\theta & \cos(\theta-\frac{2\pi}{3}) & \cos(\theta+\frac{2\pi}{3})\\ -\sin\theta & -\sin(\theta-\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3})\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}i_a\\ i_b\\ i_c\end{bmatrix}$$

Means :

$$[i_{dqo}]=[P(\theta)][i_{abc}]$$

with :

$$[P(\theta)]=[R(\theta)][Co]$$

Inverse transformation would be :

$$\begin{bmatrix}i_a\\ i_b\\ i_o\end{bmatrix}=\sqrt{\frac{2}{3}}\begin{bmatrix}\cos\theta & -\sin\theta & \frac{1}{\sqrt{2}}\\ \cos(\theta-\frac{2\pi}{3}) & -\sin(\theta-\frac{2\pi}{3}) & \frac{1}{\sqrt{2}}\\ \cos(\theta+\frac{2\pi}{3}) & -\sin(\theta+\frac{2\pi}{3}) & \frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}i_d\\ i_q\\ i_o\end{bmatrix}$$

Means :

$$[i_{abc}]=[P(\theta)]^{-1}[i_{dqo}]$$

Park transformation can be interpreted as follows:

The three-phase rotating field machine comprises three fixed coils, offset by 120 ° along the air gap and traveled by three-phase alternative currents (moved in the time of two thirds of period). By Ferraris theorem, this gives rise to a rotating magnetic field with $\omega_s=\frac{f}{p}$ speed and whose amplitude is constant over time.

Park transformation can replace the real system by a system comprising :

-two rotating windings at angular velocity $\dot{\theta}_s=\omega_s$ and traversed by currents i_d and i_q ;

- a fixed winding, traveled by homopolar current i_o.

The equivalent system gives rise to a rotating field in the air gap which is identical to that created by the three coils offset by 120 ° and crossed by three-phase currents.

Notes

4.2.2.7 Asynchronous machine equations in Park coordinates

In a reference related to rotating field, speed ω_{dq} is equal to the stator pulsation ω_s and the difference $\omega_{dq} - \omega$ is equal to $g\omega_s$, machine equations are then:

$$v_{ds} = R_s i_{ds} + \frac{d\emptyset_{ds}}{dt} - \emptyset_{qs}.\omega_s$$
$$v_{qs} = R_s i_{qs} + \frac{d\emptyset_{qs}}{dt} + \emptyset_{ds}.\omega_s$$
$$0 = R_r i_{dr} + \frac{d\emptyset_{dr}}{dt} - \emptyset_{qr}.g.\omega_s$$
$$0 = R_r i_{qr} + \frac{d\emptyset_{qr}}{dt} + \emptyset_{dr}.g.\omega_s$$

4.2.2.8 Torque calculation

Park transformation preserves the instantaneous power, we can write :

$$p(t) = v_{as}(t).i_{as}(t) + v_{bs}(t).i_{bs}(t) + v_{cs}(t).i_{cs}(t) = v_{ds}(t).i_{ds}(t) + v_{qs}(t).i_{qs}(t)$$

Substituting in the above expression the values of v_{ds} and v_{qs} from equations of this page, we get:

$$p(t) = R_s.\left(i^2{}_{ds}(t) + i^2{}_{qs}(t)\right) + \left(\frac{d\emptyset_{ds}(t)}{dt}.i_{ds}(t) + \frac{d\emptyset_{qs}(t)}{dt}.i_{qs}(t)\right) + \omega_s.(\emptyset_{ds}(t).i_{qs} - \emptyset_{qs}(t).i_{ds})$$

The first term represents Joule effect losses, the second, stored electromagnetic power and last term represents electrical power converted into mechanical power.
So, we have :

$$p_{méca}(t) = \omega_s.\left(\emptyset_{ds}(t).i_{qs} - \emptyset_{qs}(t).i_{ds}\right) = c_{méca}(t).\omega_s$$

We deduce the expression of instantaneous torque :

$$c_{méca}(t) = (\emptyset_{ds}(t).i_{qs} - \emptyset_{qs}(t).i_{ds})$$

By exploiting stator flux expressions and calling p the number of pole pairs, it is possible to establish other expressions of torque all equal:

$$\begin{cases} C_e = p.(\emptyset_{ds}.i_{qs} - \emptyset_{qs}.i_{ds}) \\ C_e = p.(\emptyset_{qr}.i_{dr} - \emptyset_{dr}.i_{qr}) \\ C_e = p.M(i_{qs}.i_{dr} - i_{ds}.i_{qr}) \\ Ce = p.\frac{M}{L_r}(\emptyset_{dr}.i_{qs} - \emptyset_{qr}.i_{ds}) \end{cases}$$

It is the latter expression that we will use in vector control presented later.

4.3. Vector control

There are several types of vector control, we only discuss in this chapter **rotor field-oriented vector control**.

Principle of vector control

Vector control, by decoupling flux and current, simplifies the torque control by making it similar to what happens to a DC machine. However, this simplification has a price : it is necessary to have sufficient computing power in order to perform necessary calculations in real time.

We have seen that the torque in transient regime is expressed in the reference dq as a cross product of current or flux. Let us resume the torque expression :

$$C_e = p.\frac{M}{L_r}(\emptyset_{dr}.i_{qs} - \emptyset_{qr}.i_{ds})$$

It is obvious that to give the torque a form exactly similar to that of a DC machine, it is necessary to eliminate the second product $(\emptyset_{qr}.i_{ds})$. To delete this product, it is sufficient to direct the *dq* reference so that the quadrature flux component will be cancelled. That is to say to choose the proper rotation angle so that the Park rotor flux is fully carried by the direct axis *d* and so $\emptyset_{qr} = 0$. So $\emptyset_r$ will be only equal to $\emptyset_{dr}$.

Notes

The torque is then:

$$C_e = p.\frac{M}{L_r}.\emptyset_{dr}.i_{qs}$$

The flux $\emptyset_{dr}$ is adjusted by acting on stator current component i_{ds} . en suite, the torque can be fixed by acting on i_{qs} component.

That's the action on i_{ds} which allows to adjust the flux. Thus, considering the rotor short-circuited, we have :

$$\begin{cases} \emptyset_{dr} = L_s i_{dr} + M i_{ds} \\ 0 = R_r i_{dr} + \frac{d\emptyset_{dr}}{dt} \end{cases}$$

Known that the flux $\emptyset_{qr}$ is cancelled in this command, we deduce by substitution :

$$\emptyset_{dr} + \frac{L_r}{R_r}.\frac{d\emptyset_{dr}}{dt} = M.i_{ds}$$

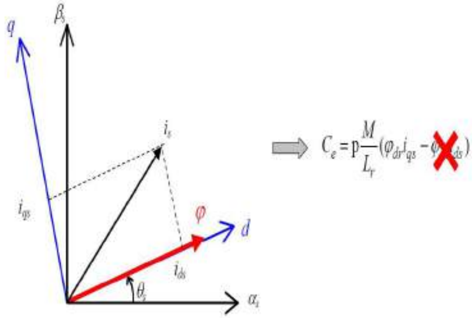


Figure 4.3 : Vector control principle.

Once torque regulation is mastered, we can add an external regulation to speed control. It is then a cascade control and loops are nested (imbricated) one within the other. It is obvious that to increase the speed, it is necessary to impose a positive torque and to decrease it, it is necessary to impose a negative one. It becomes clear that speed controller output must be the torque. This reference torque must be imposed by current application; It is the role of currents regulators.

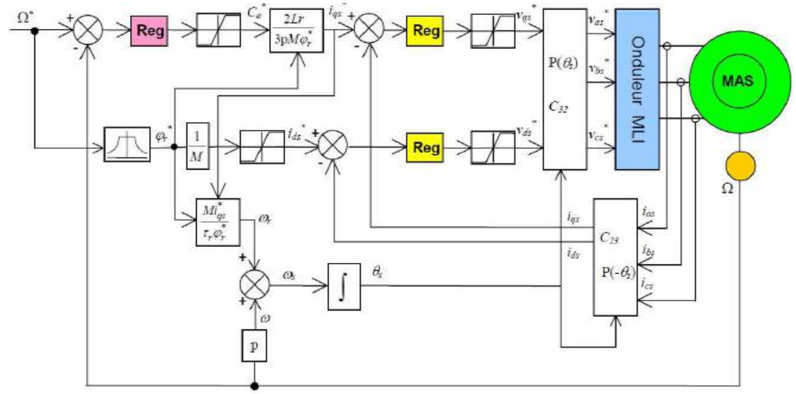


Figure 4.3 : Scheme of induction machine speed regulation with rotor field-oriented vector control.

Command implementation

To be able to command (control) components, it is necessary to return in the ABC reference, thus to know d axis position. This position is identified by the angle θ_s that checks : $\theta_s = \theta + \theta_r$.

The angle θ is measured by a position sensor and θ_r can be obtained by integrating rotor (pulsation) angular magnitudes ω_r .

From the relationships:

$$0 = R_r i_{qr} + \omega_r \emptyset_{dr}$$

$$\emptyset_{qr} = 0 \Rightarrow L_r i_{qr} + M i_{qs} = 0$$

Pulsation of rotor variables spells :

$$\omega_r = -\frac{M.R_r}{L_r}.\frac{i_{qs}}{\emptyset_{dr}} = -\frac{M}{\tau_r}.\frac{i_{qs}}{\emptyset_{dr}}$$

It can therefore be assessed from the estimated value of i_{qs} and calculated value of $\emptyset_{dr}$.

Notes

Park machine behaves as a direct current machine with separate excitation :
- The field flux is $\emptyset_{dr}$, since we impose $\emptyset_{qr} = 0$;
- The equivalent current of the armature current is i_{qs}.

We then have two action variables as in the case of a DC machine. One strategy is to leave i_{ds} component constant, means to set its reference so as to impose a nominal flux in the machine. i_{ds} current regulator is then responsible for maintaining it constant and equal to the reference i^*_{ds}.

The flux in the machine being constant, we can impose variations in torque acting on current i_{qs}. If the machine has to be accelerated, thus its speed increased, we impose a poisitive current reference i^*_{qs}. It is i_{qs} regulator which is going to impose this reference current on the machine.

We can also automate the control of this current reference i^*_{qs} by connecting it to speed controller output. It is the latter which will drive the reference torque (and therefore i^*_{qs}) since it will act at best so as to control the speed at a set value Ω^*.

Figure 4.3 summarizes this regulation, it is a drawing of induction motor vector control with speed control and regulation of the two currents i_{ds} and i_{qs}. These two currents are regulated by two currents loops whose outputs are reference voltages v^*_{ds} and v^*_{qs} in *dq* reference.

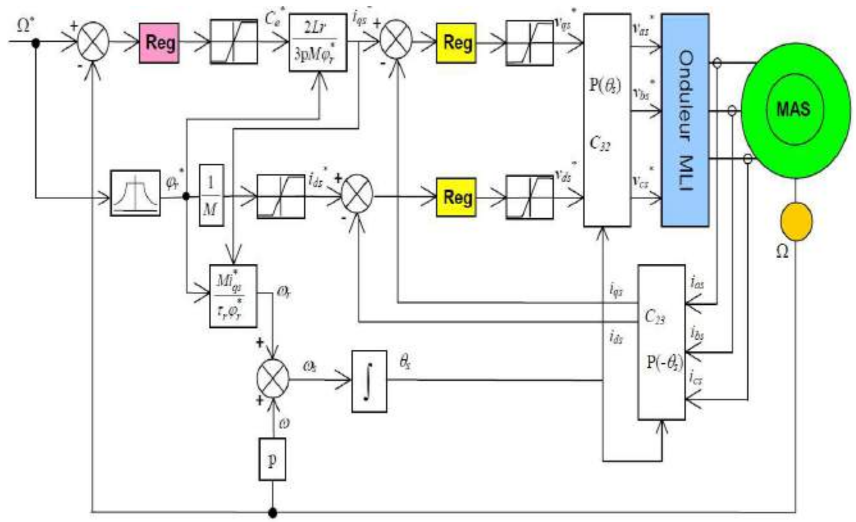


Figure 4.3 : Scheme of induction machine speed regulation with rotor field-oriented vector control.

The diagram in Figure 4.3 has three controllers:

Speed regulator : it acts on the torque to control the speed, its output is torque reference.

i_{qs} current regulator : it acts on voltage reference v^*_{qs} to adjust the current i_{qs}.

If we look more closely at the diagram, we see that there is a ratio between the torque reference and the current reference i^*_{qs}. This coefficient takes into account the flux value (see torque formula) but also a 2/3 factor that depends on three-phase - two-phase transformation. In this diagram, the presence of this factor 2/3 is due to the choice of Clarke transformation.

i_{ds} current regulator : it acts on voltage reference v^*_{ds}. Regulating the current to a constant value ensures a constant rotor flux : $\emptyset_r = \frac{M}{1+p\tau_r}.i_{ds}$ with $\tau_r = \frac{L_r}{R_r}$ the rotor time constant and p the variable of Laplace transformation. We see while in permanent regime $\emptyset_r = M.i_{ds}$.

Besides, two transformations are important:

Inverse Park transformation: which, from biphasic voltages (v^*_{ds}, v^*_{qs}) in *dq* reference, allows to calculate the three phase voltages $v^*_{as}, v^*_{bs}, v^*_{cs}$ to impose on the machine via PWM inverter;

Direct Park transformation: which, from the three line currents of the machine, allows to calculate the two-phase currents (i_{ds}, i_{qs}) to be regulated in the dq reference.

Notes

These two transformations require the computation of θ_s angle. The block responsible for this calculation uses measured speed and slip pulsation ω_r.

Under rotor field-oriented vector control, slip pulsation is calculated by $\omega_r = \frac{i_{qs}}{\tau_r i_{ds}}$ or or by using references instead measures. Thus the calculation of the angle of direct and reverse transformations can be done by summing the slip pulsation with electric speed, what gives the stator pulsation. Then by integrating it, we obtain θ_s.

$$\boldsymbol{\theta_s = \int \omega_s \, dt = \int (p\Omega + \frac{i^*_{qs}}{\tau_r i^*_{ds}})\mathrm{dt}}$$

This gives the general scheme to be implemented on a digital controller (DSP or micro-controller).

Every period of inverter operation, command has to open or close power switches (IGBT or other one) so as to create in the electric machine a resultant magnetic field whose module and direction are optimal to ensure required speed and torque. The computer, which will act on switch command, must have some information to make calculations and particularly:

- rotor position - rotor speed.

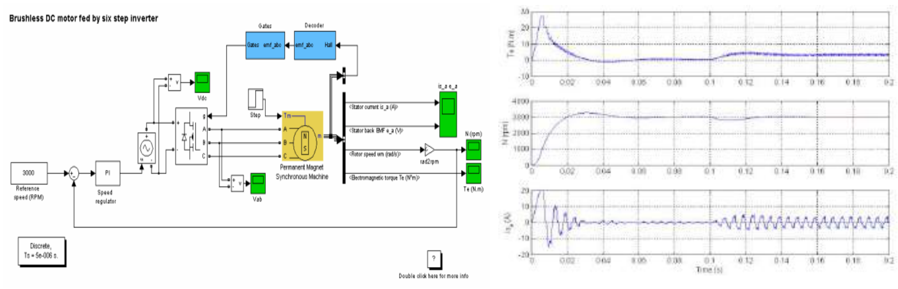
This information is obtained by using a sensor of position or speed. Nevertheless, it is possible to reconstitute this information with more or less precision by means of electric information such as the knowledge of currents. **This is called sensorless vector control**.

Notes

Notes

Notes

Chapter 5 : Dynamic modeling of the synchronous machine (study in any regime)



5.1. Introduction

Synchronous machines have a higher power density than asynchronous machines and rotor flux is known. So, it is easier to control its torque.
The progress made in magnets manufacture, even those based on metal or rare earth alloys, makes the use of PMSM today go increasing.

5.2. Modeling

5.2.1.Representation in two-phase reference

For PMSM, this reference will be linked with rotor axis in the direction of magnetic induction.

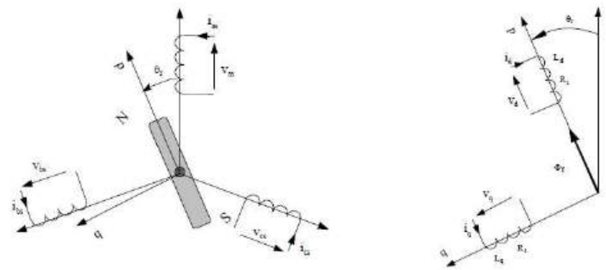


Figure 5.1 : Representation of PMSM in three-phase reference (a,b,c) or two-phase reference (d-q).

In this new reference, we note :

$L_d(H)$: Armature equivalent inductance on the d-axis.
$L_q(H)$: Equivalent inductance of the armature on the q-axis.
$R_s(\Omega)$: Equivalent resistance of stator windings.
P : Number of pole pairs.
f : Fluid friction coefficient.
J : Rotor inertia.

Note that here the PMSM is reduced to a machine with a pair of pole, θ_r angle corresponds to the actual angle of the rotor multiplied by the number of pole pairs P.

5.2.2. Machine Park equations

If we consider a sinusoidal distribution of magnetic induction and neglecting the saturation phenomena in iron, we have in the reference d-q the following relations :

Voltages equations :

$$V_d = R_s . I_d + \frac{d\phi_d}{dt} - \omega_r . \phi_q$$

$$V_q = R_s . I_q + \frac{d\phi_q}{dt} + \omega_r . \phi_d$$

Notes

Flux equations :

$$\emptyset_d = L_d I_d + \emptyset_f \qquad \emptyset_f : \text{inductor flux}$$
$$\emptyset_q = L_q I_q$$

Electromagnetic torque expressions :

$$C_{em} = P.\left(\emptyset_d.I_q - \emptyset_q.I_d\right)$$
$$Cem = P.\left(I_d.I_q.\left(L_d - L_q\right) + \emptyset_f.I_q\right)$$

Special case for the machine with smooth poles ($L_d = L_q$) :

$$C_{em} = P.\emptyset_f.I_q$$

In this case, as the current I_d is not involved in torque equation, the minimum of joule losses is reached for I_d zero.

5.2.3.Equations in the reference α, β bound to the stator

Voltages:

$$V_\alpha = R_s.I_\alpha + L_s\frac{dI_\alpha}{dt} - \omega_r.\emptyset_f.\sin(\theta_s)$$
$$V_\beta = R_s.I_\beta + L_s\frac{dI_\beta}{dt} + \omega_r.\emptyset_f.\cos(\theta_s)$$

Flux:

$$\emptyset_{s\alpha} = L_s.I_\alpha + \emptyset_f.\cos(\theta_s)$$
$$\emptyset_{s\beta} = L_s.I_\beta + \emptyset_f.\sin(\theta_s)$$

Torque:

$$C_{em} = p(\emptyset_\alpha.I_\beta - \emptyset_\beta.I_\alpha)$$

Currents and fluxes components are sinusoidal.

5.3. Decoupling currents I_d and I_q

To command this motor, it is imperative to control the torque. This latter only depends on stator currents components in the d-q reference. So It is necessary to master these.

It is clear that les courrents I_d and I_q depend simultaneously on input variables V_d and V_q. In order to implement single variable controls, we go from equations governing motor dynamic regime look for one against not linear reaction which decouples the system.

We can write:

$$V_d = R_s.I_d + L_d.\frac{dI_d}{dt} - \omega_r.L_q.I_q$$
$$V_q = R_s.I_q + L_q.\frac{dI_q}{dt} + \omega_r.L_d.I_d + \omega_r.\emptyset_f$$

To decouple the evolution of currents I_d and I_q relative to commands, let's define compensation terms E_d and E_q as :

For the first component of the stator current, we have :

$$V_d + \omega_r.L_q.I_q = R_s.I_d + L_d.\frac{dI_d}{dt} = V'_d = V_d + E_d$$

with : $$E_d = \omega_r.L_q.I_q = \omega_r.\emptyset_q$$

For the second component, we get :

$$V_q - \omega_r.L_d.I_d - \omega_r.\emptyset_f = R_s.I_q + L_q.\frac{dI_q}{dt} = V'_q = V_q - E_q$$

with : $$E_q = \omega_r.L_d.I_d + \omega_r.\emptyset_f = \omega_r.\emptyset_d$$

With new entries (inputs) V'_d and V'_q, we can from differential equations define two mono variable transmittances:

$$\frac{I_d(p)}{V'_d(p)} = \frac{1}{R_s + L_d.p}$$

$$\frac{I_q(p)}{V'_q(p)} = \frac{1}{R_s + L_q.p}$$

Notes

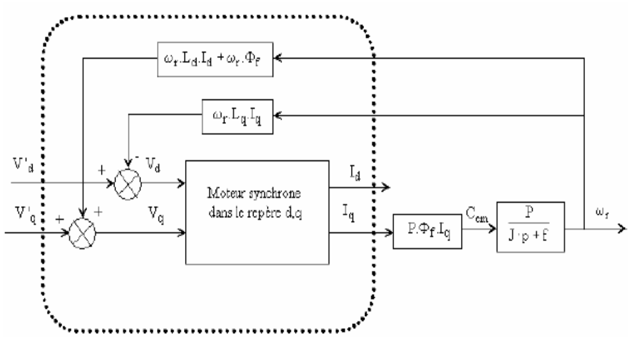


Figure 5.2 : Decoupling of magnet synchronous machine.

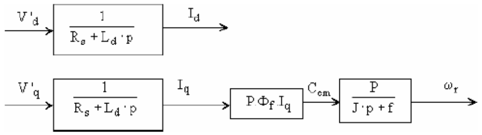


Figure 5.3 : Behavior of the SM with the decoupling.

5.4. Magnet synchronous machine control

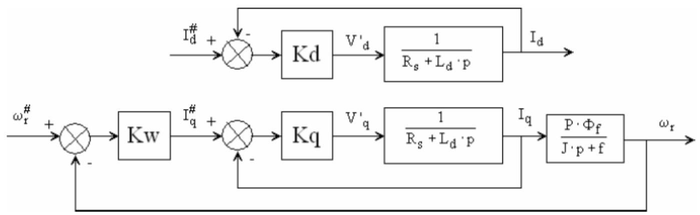


Figure 5.4 : Command loops.

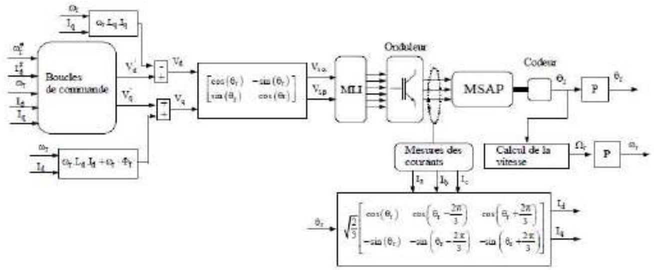
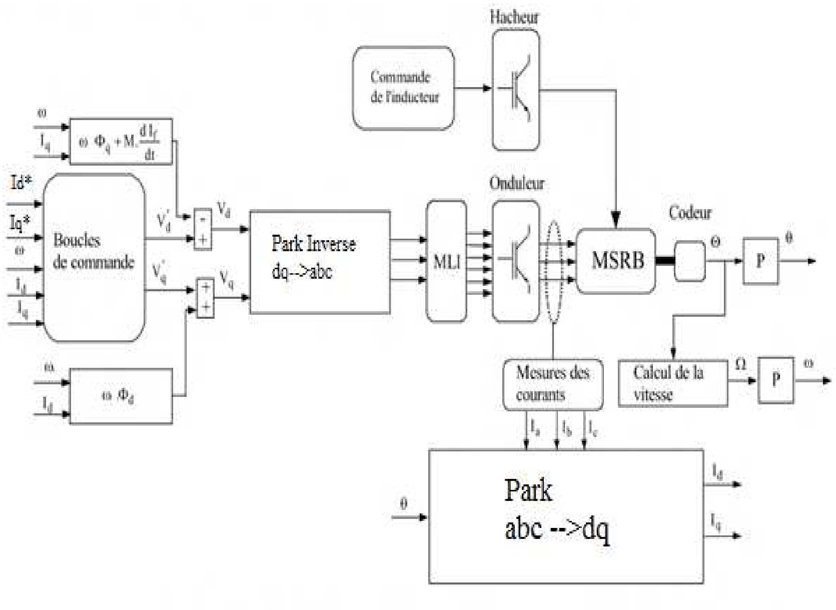


Figure 5.5 : Technological scheme.



Notes

Chapter 6 :
Power converters associated with electrical machines

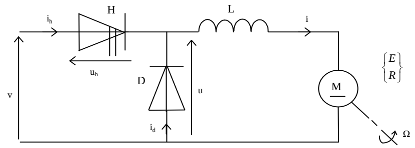


Figure 6.1 : Chopper and DC machine.

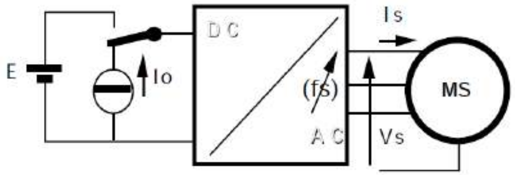


Figure 6.2 : SM alimentation with variable frequency.

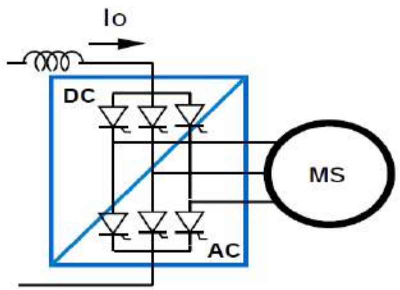


Figure 6.3 : SM powered by a three-phase Graetz bridge of thyristors .

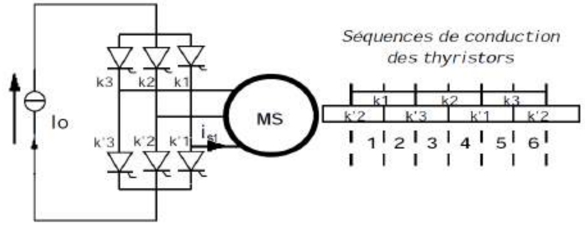


Figure 6.4: Switching currents in SM phases.

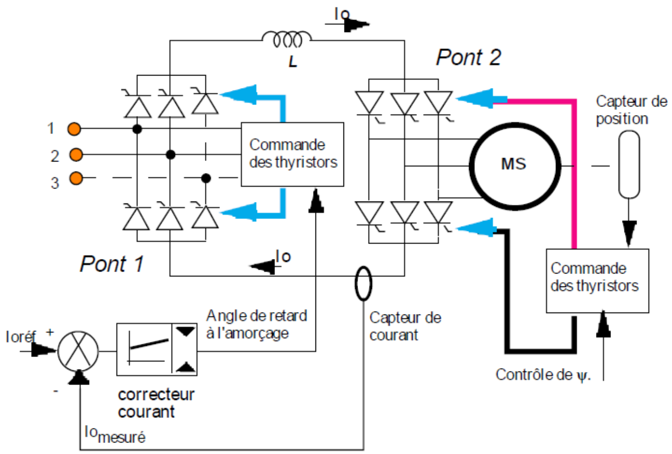


Figure 6.5:Together converter-machine.

Notes

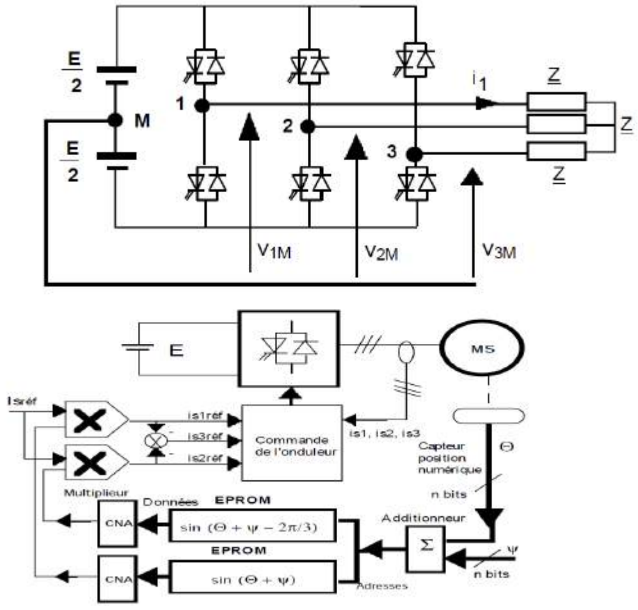


Figure 6.5 : A synchronous machine fed by a PWM voltage inverter.

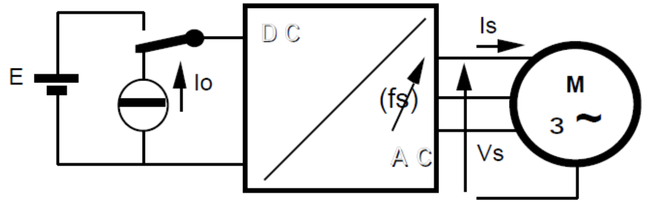


Figure 6.6: Asynchronous machine alimentation with variable frequency.

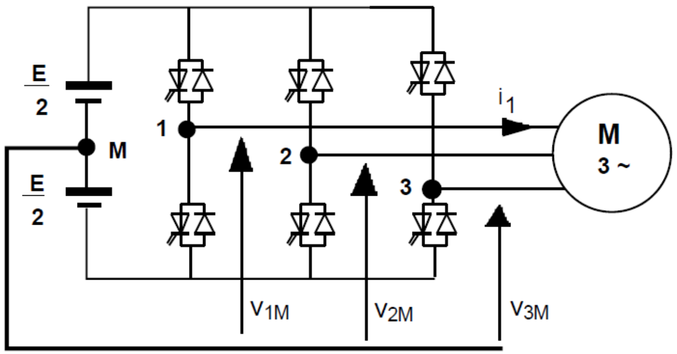


Figure 6.7:Induction motor-PWM inverter.

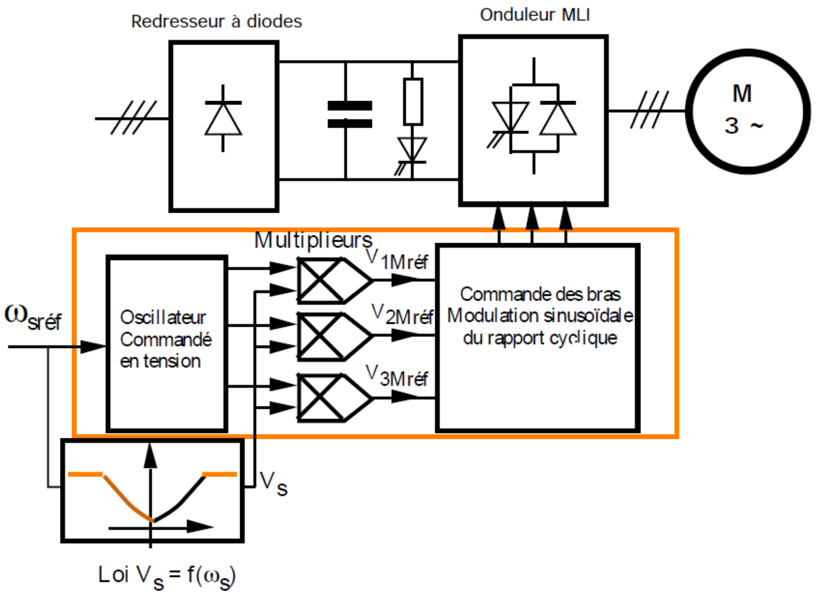


Figure 6.8:Principle of PWM inverter control.

Notes