CHAPTER THREE

Three-Phase AC POWER

Learning Outcomes:

1.Describe the reasons for, and the generation of the three-phase supply.

2.Distinguish between star (3 and 4-wire) and delta connections.

3.State the relative advantages of three-phase systems compared with single-phase-systems.

4.Solve three-phase circuits in terms of phase and line quantities and the power developed in three-phase balanced loads.

5.Measure power dissipation in both balanced and unbalanced three-phase loads, using the 1, 2 and 3-wattmeter methods, and hence determine load power factor

WHY?

Three-phase ac power is one of the most common forms of electric power distribution worldwide.

Many countries use three-phase ac power for power distribution since it is simpler, cheaper, and more efficient than single-phase ac power.

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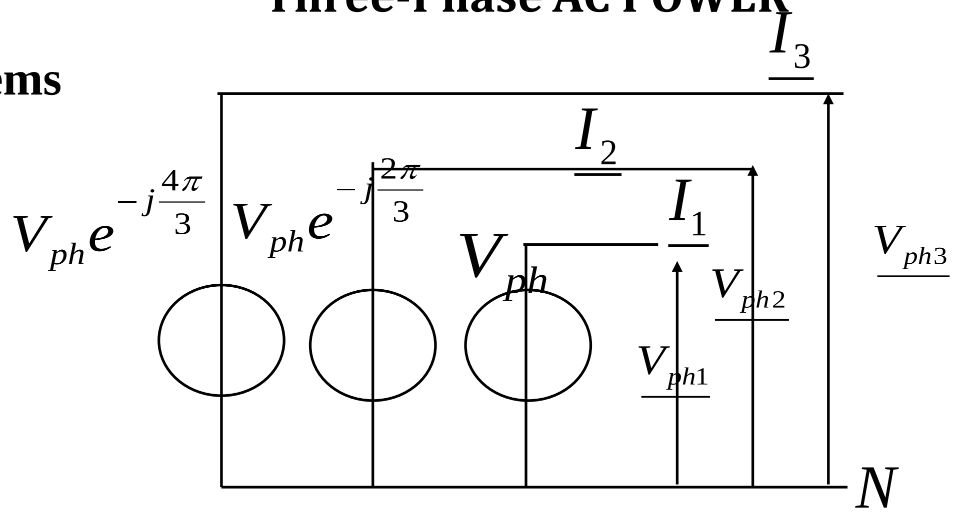
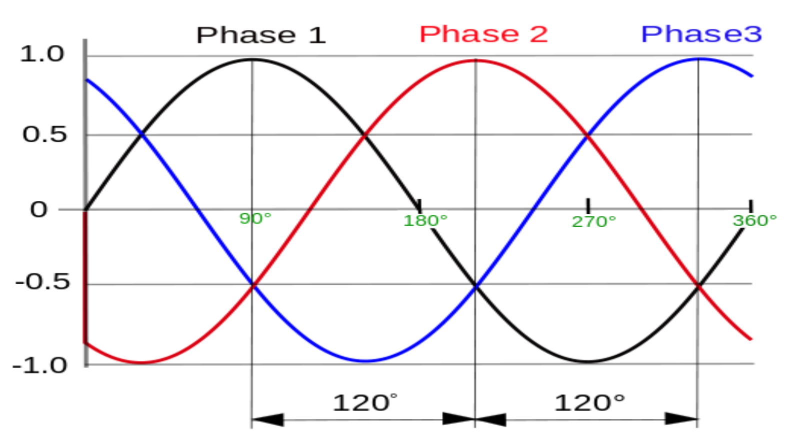
Three Phase Systems



$$V_{ph1} = V_{ph}\sqrt{2}\sin(\omega t)$$

$$V_{ph2} = V_{ph}\sqrt{2}\sin(\omega t - \frac{2\pi}{3})$$

$$V_{ph3} = V_{ph}\sqrt{2}\sin(\omega t - \frac{4\pi}{3})$$

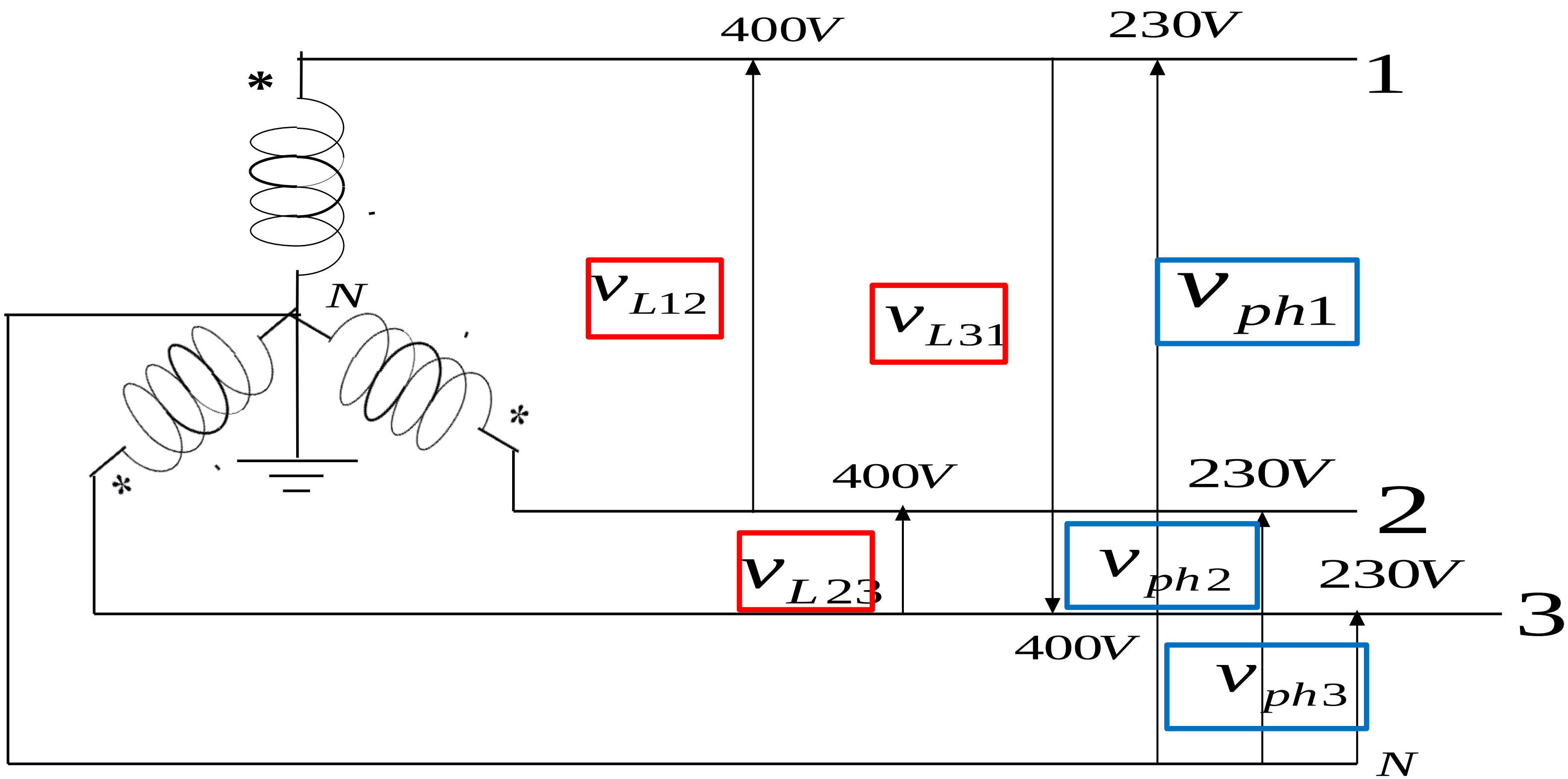


$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

$$\underline{V}_{ph1} + \underline{V}_{ph2} + \underline{V}_{ph3} = 0$$

Wye and delta configurations

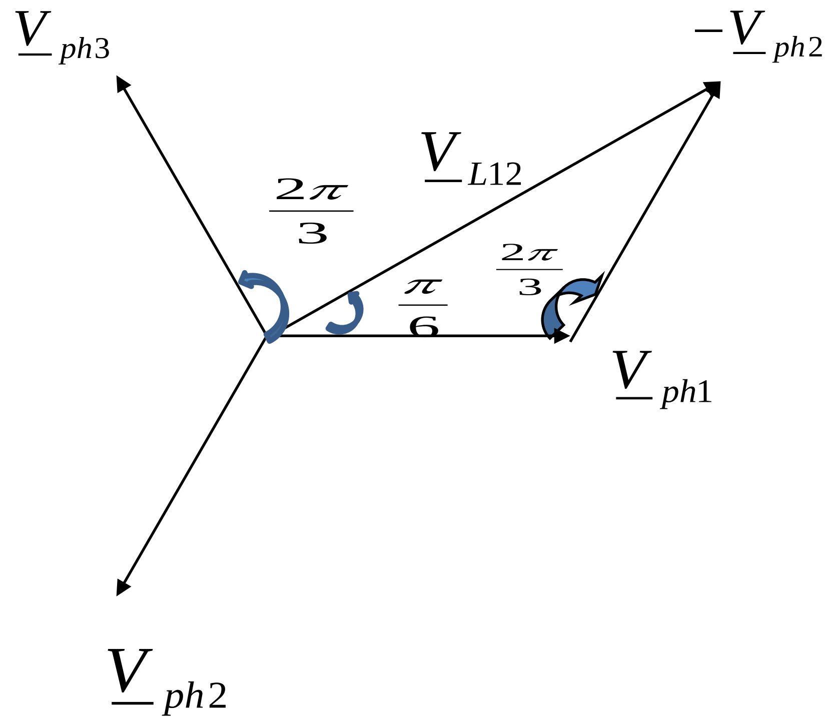
Wye configuration Y (Star configuration)



The three phase voltages are: $\underline{V_{ph1}}$ $\underline{V_{ph2}}$ $\underline{V_{ph3}}$

The three line voltages are: $\underline{V_{L12}}$ $\underline{V_{L23}}$ $\underline{V_{L31}}$

Phasor Diagram



$$\underline{V}_L = U = \sqrt{3} V_{ph} e^{j\frac{\pi}{6}}$$

$$V_L = 2V_{ph}\cos\frac{\pi}{6} = 2V_{ph}\frac{\sqrt{3}}{2} = V_{ph}\sqrt{3}$$

$$\boxed{V_L = U = V_{ph}\sqrt{3}}$$

$$P = \sqrt{3} V_L I_L \cos\varphi = \sqrt{3} U I_L \cos\varphi$$

$$Q = \sqrt{3} V_L I_L \sin\varphi = \sqrt{3} U I_L \sin\varphi$$

$$S = \sqrt{3} V_L I_L = \sqrt{3} U I_L$$

Exercise 1

The peak value of the line voltage in all three phases is 563, 38 V. Calculate:

a) The effective phase voltage (voltage between a phase and neutral line);

b) The effective line voltage that can be taken between two phase lines

Answers

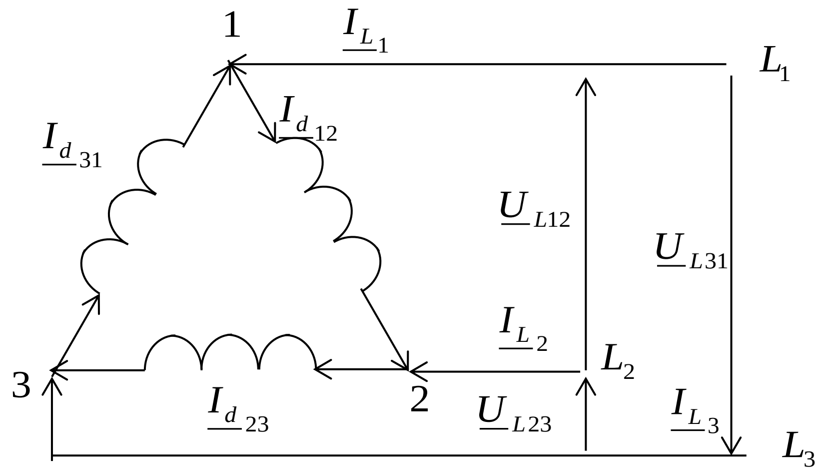
a) The effective phase voltage (voltage between a phase and neutral line);

$$V_{ph} = \frac{V_{L-peak}}{\sqrt{2} \times \sqrt{3}} = \frac{563{,}38}{\sqrt{2} \times \sqrt{3}} = 230V$$

b) The effective line voltage that can be taken between two phase lines

$$V_L = U = \frac{V_{L-peak}}{\sqrt{2}} = \frac{563{,}38}{\sqrt{2}} \cong 400V$$

Delta configuration D

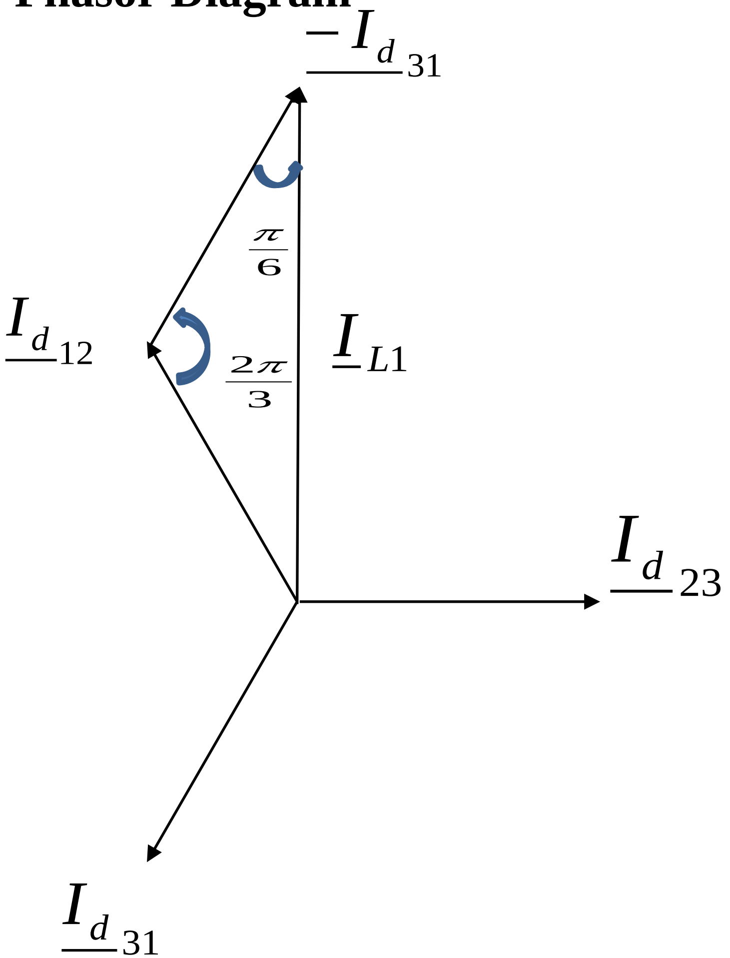


$$P = \sqrt{3} V_L I_L \cos\varphi = \sqrt{3} U I_L \cos\varphi$$

$$Q = \sqrt{3} V_L I_L \sin\varphi = \sqrt{3} U I_L \sin\varphi$$

$$S = \sqrt{3} V_L I_L = \sqrt{3} U I_L$$

Phasor Diagram

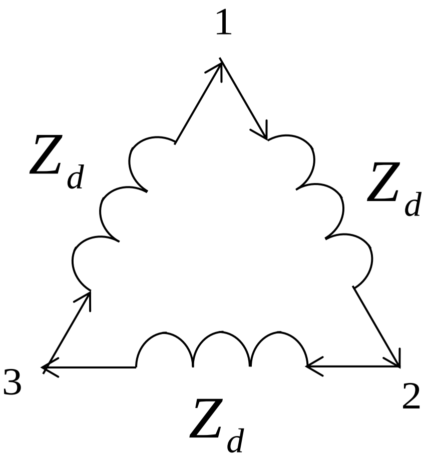


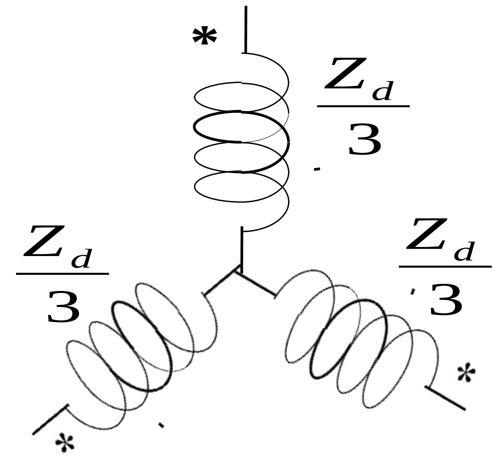
$$\underline{I}_L = I_d e^{-j\frac{\pi}{6}}$$

$$I_L = 2I_d \cos\frac{\pi}{6} = 2I_d \frac{\sqrt{3}}{2} = I_d\sqrt{3}$$

$$I_L = I_d\sqrt{3}$$

The star delta transformation





$$Z_s = \frac{Z_d}{3}$$

Per-Phase Analysis in Balanced 3-Phase Circuits

- **To do per phase analysis**

1. Convert all 3ϕ load/sources to equivalent Y's
2. Solve phase "a" independent of the other phases
3. Total system power $S = 3\ V_a\ I_a^*$

Exercise 2. Suppose it is necessary to build a resistive heater to deliver 6kW, to be made of three elements which may be connected in either wye or delta. Each of the three elements must dissipate 2000 W. Find R_Y and R_D

Solution

Thus, since $P = \frac{V^2}{R}$

The wye connected resistors would be:

$$P = \frac{V_{ph}^2}{R_Y} \rightarrow R_Y = \frac{230^2}{2000} = 26{,}45\Omega$$

While the delta connected resistors would be:

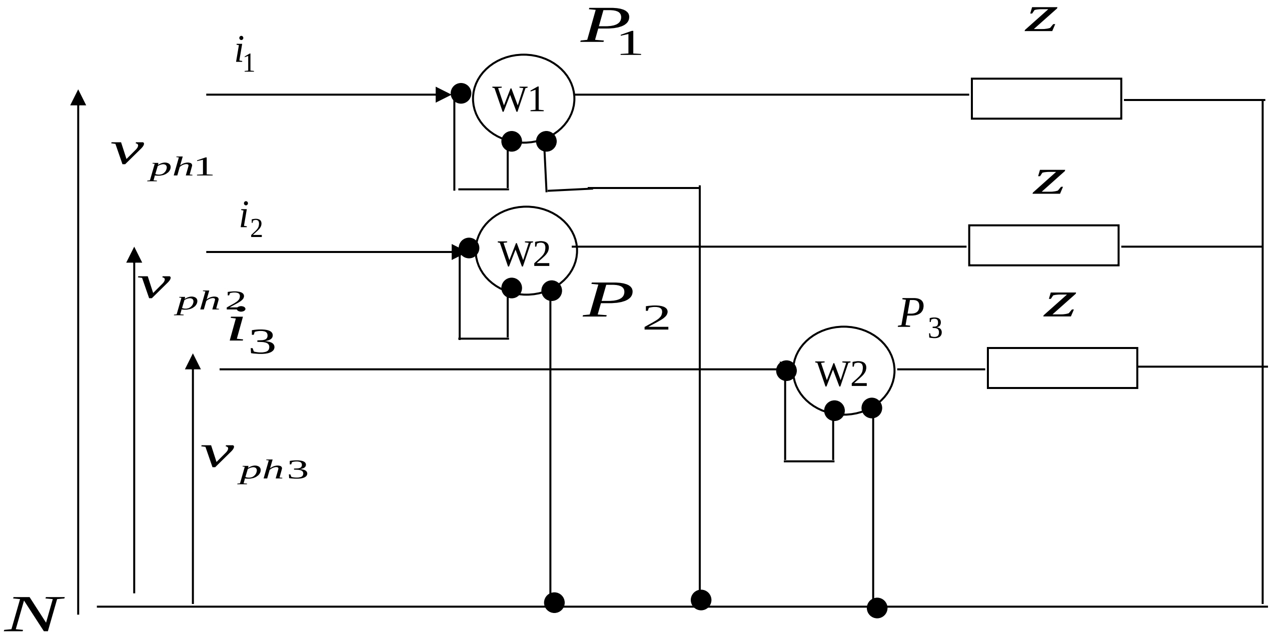
$$P = \frac{V_L^2}{R_\Delta} = \frac{U^2}{R_\Delta} \rightarrow R_\Delta = \frac{400^2}{2000} = 80\Omega$$

As is suggested by this example, wye and delta connected impedances are often directly equivalent.

$$Z_D = 3 \times Z_Y \rightarrow R_D = 3 \times R_Y$$

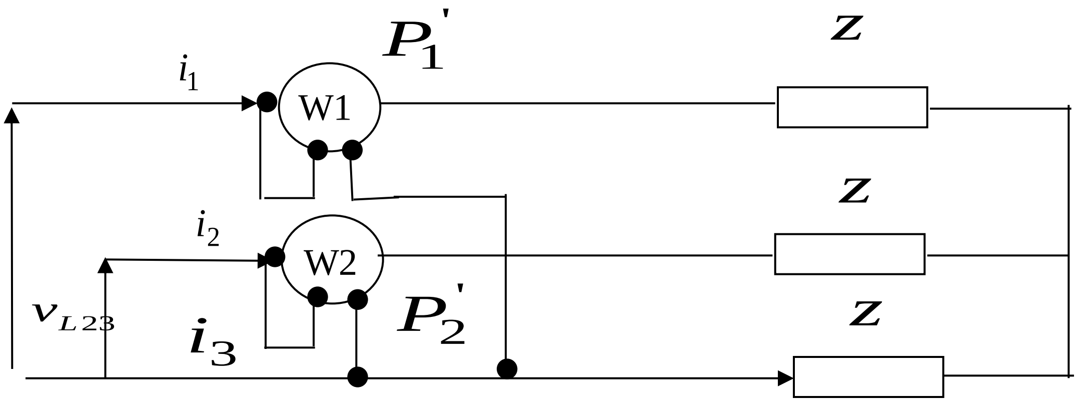
Three-Phase Power Measurements

Three-wattmeter configuration



$$P = P_1 + P_2 + P_3$$

Two-wattmeter configuration



$$P = P_1^{'} + P_2^{'}$$

$$Q = \sqrt{3}\left(P_1^{'} - P_2^{'}\right)$$

Advantages of 3-Phase Systems

- Can transmit more power for same amount of wire (twice as much as single phase)
- Torque produced by 3ϕ machines is constant
- Three phase machines use less material for same power rating
- Three phase machines start more easily than single phase machines

REVIEW QUESTIONS

1. In a balanced wye-connected circuit, the
a. Line voltages and currents equal the load values.
b. Line voltage is 3 times greater than the phase voltage.
c. Line voltage is 3 times smaller than the phase current.
d. Line current is 3 times greater than the phase current.

2. In a balanced delta connected circuit, the
a. Line voltages and currents equal the load values.
b. Line current is 3 times smaller than the phase current.
c. Line current is $\sqrt{3}$ times greater than the phase current.
d. Line voltage is 3 times greater than the phase voltage.

3. What is the line-to-neutral (phase) voltage in a balanced wye-connected circuit when the line-to-line voltage is 400V?
a. 346V
b. 600V
c. 200V
d. 230V

Solution

1) a;
2) c;
3) d;

4. What is the line current in a balanced delta-connected resistive load when the load current through each branch is 10A?
a. 27.3A
b. 17.3A
c. 11.6A
d. 5.8A
5. In a three-phase balanced circuit, the active power can be determined using two wattmeters connected according to the:
a. Single-phase wattmeter method
b. Three-phase wattmeter method
c. Two-wattmeter method
d. Apparent power method
6. The formula for total active power in a three-phase balanced circuit is:
a. $P_{active} = \sqrt{3}(E_{phase} \times I_{phase} \times \cos\varphi)$
b. $P_{active} = \sqrt{3}(E_{line} \times I_{phase} \times \cos\varphi)$
c. $P_{active} = \sqrt{3}(E_{phase} \times I_{line} \times \cos\varphi)$
d. $P_{active} = \sqrt{3}(E_{line} \times I_{line} \times \cos\varphi)$

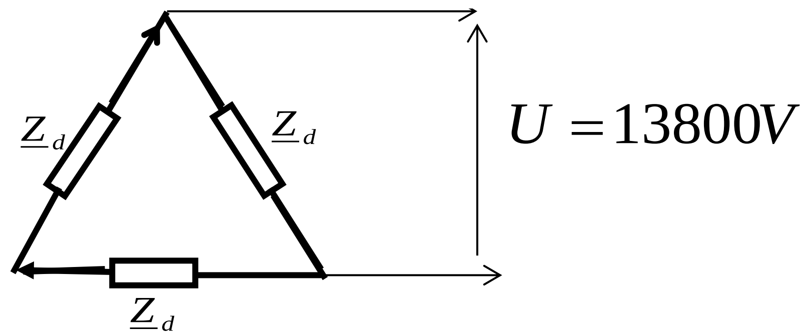
Solution

4) b;
5) c;
6) d

Exercise 3.

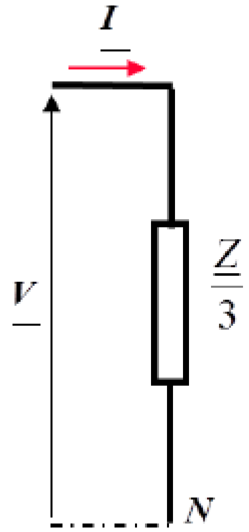
A circuit is set up as shown below. The frequency of the source is 50Hz. Given $\underline{Z} = (240 + j70)\Omega$.

1. Convert the load to Y and work with one phase
2. Calculate *P, Q, S*



Solution

1. Convert the load to Y and work with one phase



2. Calculate *P, Q, S*

$$\underline{Z}_Y = \frac{\underline{Z}_D}{3} \qquad \frac{\underline{Z}}{3} = (80 + j23{,}33)\Omega$$

$$P_{1\phi} = R \times I^2 \qquad P_{1\phi} = R \times I^2 = R \times \frac{V^2}{\left(\frac{Z}{3}\right)^2} = 80 \times \frac{\left(\frac{13800}{\sqrt{3}}\right)^2}{80^2 + 23.33^2} = 731299.68W$$

$$P_{3\phi} = 3 \times P_{1\phi} \cong 2194kW$$

$$Q_{1\phi} = X \times I^2$$

$$Q_{1\phi} = X \times I^2 = X \times \frac{V^2}{\left(\frac{Z}{3}\right)^2} = 23.33 \times \frac{\left(\frac{13800}{\sqrt{3}}\right)^2}{80^2 + 23.33^2} = 213265.27VAR$$

$$Q_{3\phi} = 3 \times Q_{1\phi} \cong 640kVAR$$

$$S_{3\phi} = \sqrt{\left(P_{3\phi}^2 + Q_{3\phi}^2\right)} \cong 2285kVA$$


The End