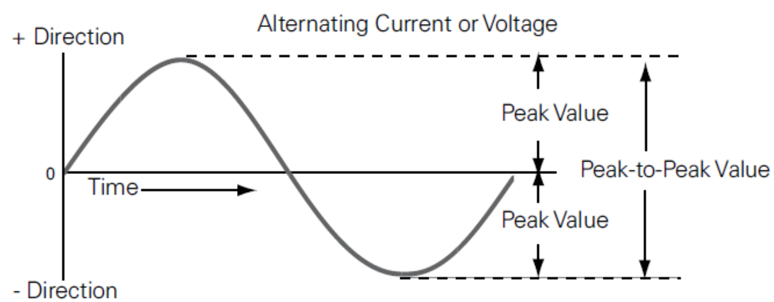
CHAPTER TWO

Alternative current (AC) Sine Wave

Single-phase AC power



WHY AC? Importance of AC

Electric power is generated, transmitted, distributed and consumed in AC

≈ 90% of the total electric power is consumed as AC

Learning Outcomes:

After successfully studying this course, students will be able to:

1. Systematically obtain the equations that characterize the performance of an electric circuit as solving single phase circuits in sinusoidal steady state

2. Acquire skills in using electrical measuring devices.

3. Simple AC series and parallel circuits are analysed using phasor and fundamental circuit laws.

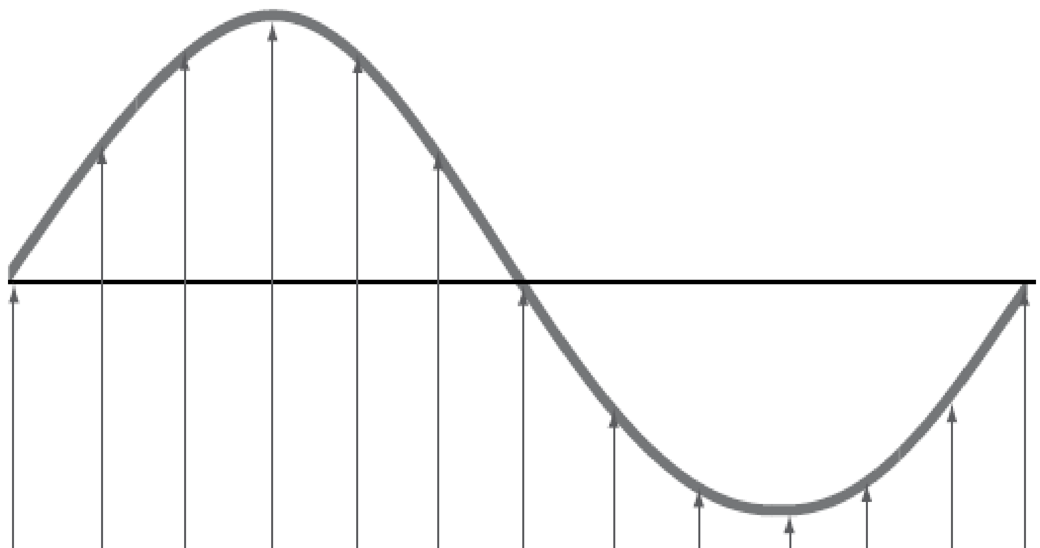
4. Perform calculations with power in AC circuits. Power and power factor for simple AC circuits are calculated

Why Single-phase AC power?

- Single phase is used primarily only in low voltage, low power settings, such as residential and some commercial

Instantaneous Value

Instantaneous Value of Alternating Current or Voltage



Angle	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
Sin θ	0.000	0.500	0.866	1.000	0.866	0.500	0.000	-0.500	-0.866	-1.000	-0.866	-0.500	0.000

Instaneous voltage $$v(t) = V_{peak} \sin\theta$$

Example: if Vpeak = 325, 26 V, at 150 degrees, then $$v(t) = 0{,}5 \times V_{peak} = 162{,}63V$$

Definitions

Voltage (V)

The force required to make electricity flow through a conductor.

The unit of measurement for voltage is the volt.

Frequency (f)

The number of cycles per second of voltage or current

The unit of measurement for frequency is the is hertz

Period (T)

$$T = \frac{1}{f}$$

The unit of measurement for period is the is second

Amplitude

The amplitude is the range of variation.
peak value,
peak-to peak value,
and effective value

The **peak value** of a sine wave is the **maximum** value

The **effective value** of AC is defined in terms of an equivalent heating effect when compared to DC

Sine definition

exercise

What is the amplitude, pulse wave, frequency, period and angle at the origin of $v(t) = 325\sin(314t + 1)$ V_p, ω, T, f, ϕ

$$V_p = 325V$$

$$\omega = 314 rad/\sec$$

$$f = \frac{1}{T} = 50Hz$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{314} \approx 20ms$$

$$\phi = 1rad$$

Root Mean Square Value (RMS)

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = \frac{V_{peak}}{\sqrt{2}}$$

Average value = 0

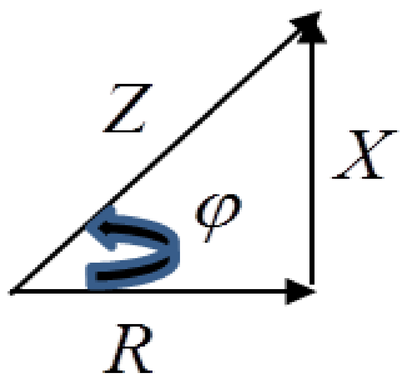
$$< v(t) >= \frac{1}{T}\int_0^T v(t).dt = 0$$

Impedance

Total opposition to current flow in an AC circuit that contains both reactance and resistance is called impedance, designated by the symbol Z. Just like resistance, reactance and impedance are expressed in Ohm

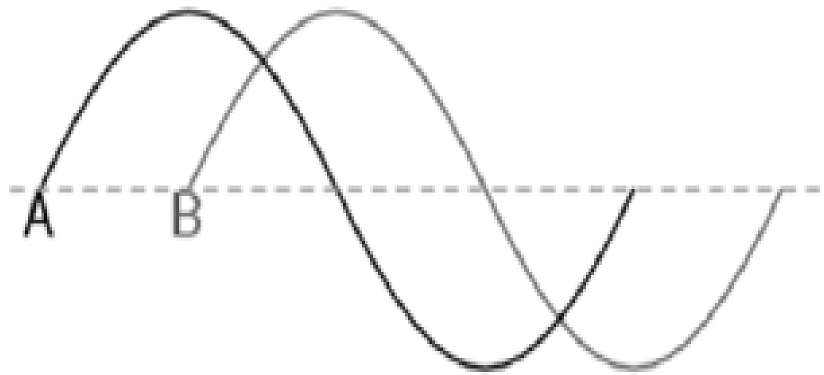
$$\underline{Z} = R + jX$$

Impedance triangle



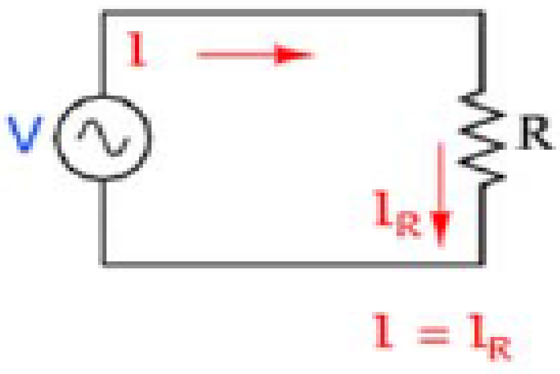
$$|\underline{Z}| = \sqrt{R^2 + X^2} \qquad R = |\underline{Z}|\cos\varphi \qquad X = |\underline{Z}|\sin\varphi \qquad \varphi = \arctan\frac{X}{R}$$

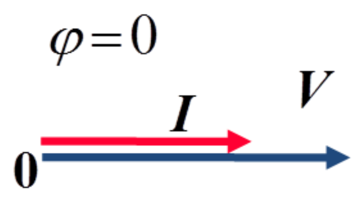
PHASE IN AC CIRCUITS

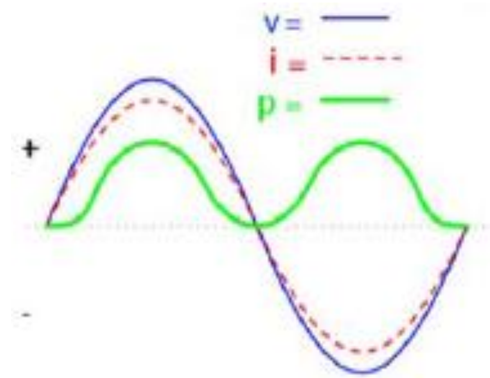


Phase shift of 90 degrees:
A leads B
B lags A

AC PURE RESISTIVE CIRCUITS





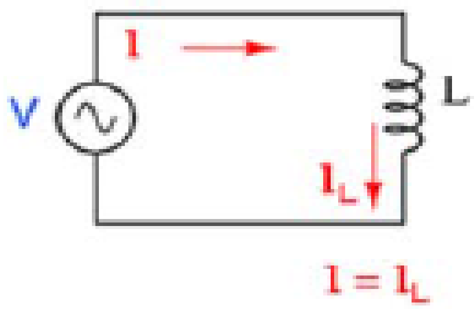


Voltage and current are “in phase”

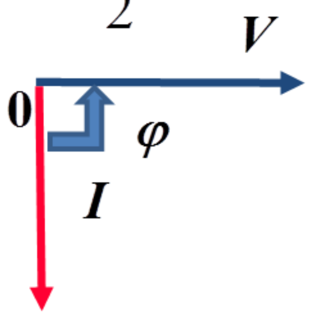
$V = RI$ $\qquad P = RI^2$ $\qquad Q = 0$

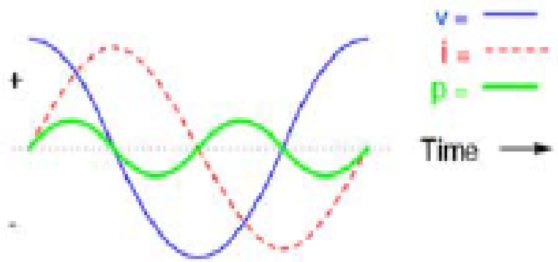
- Instantaneous AC power is always positive.
- $\varphi = 0$ <u>Phase shift</u> between voltage and current $\cos\varphi = 1$

AC PURE INDUCTIVE CIRCUITS



$$\varphi = +\frac{\pi}{2} > 0$$

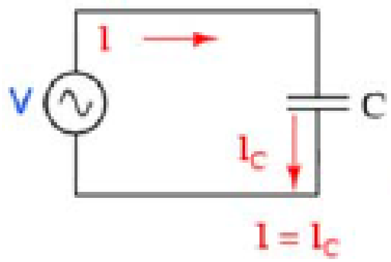


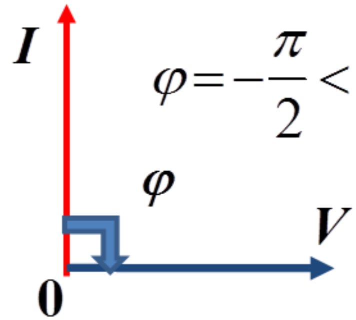


$$\underline{V} = jL\omega \underline{I} \qquad Q = L\omega I^2 \qquad P = 0$$

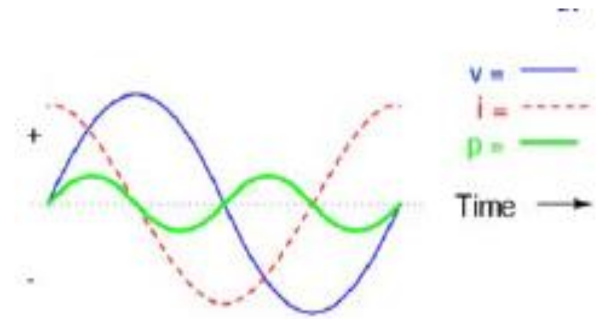
Inductor current lags inductor voltage by 90º
Instantaneous AC power is positive or negative

AC PURE CAPACITIVE CIRCUITS





$$\varphi = -\frac{\pi}{2} < 0$$



$$\underline{V} = -j\frac{1}{C\omega}\underline{I} \qquad Q = -C\omega V^2 \qquad P = 0$$

Review 1

1. A ____________ is the graphic representation of AC voltage or current values over time.
2. An AC generator produces ______________ cycle(s) per revolution for each pair of poles.
3. The instantaneous voltage at 240 degrees for a sine wave with a peak voltage of 150 V is _________ V.
4. The effective voltage for a sine wave with a peak voltage of 150 V is _________ V.

Answers

1)Sine wave;

2) one;

3) -129.9;

4) 106.05.

Exercise1

A circuit is set up as shown below. The frequency of the source is 50Hz

Calculate for the circuit:

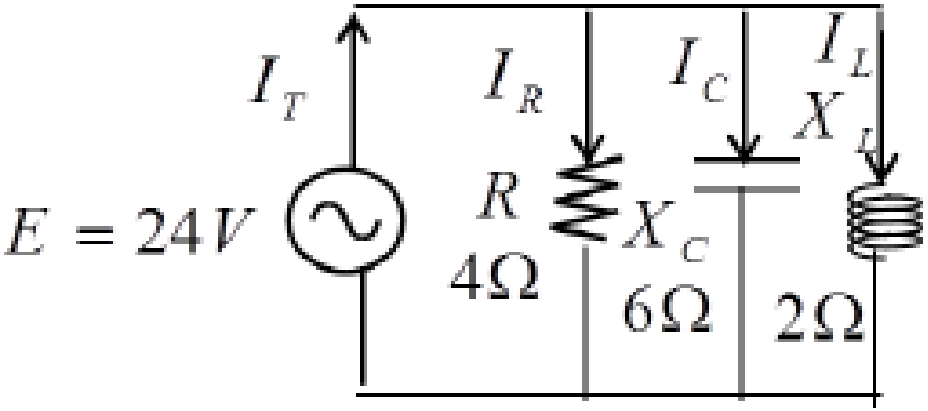
a) The current $\underline{I_R}$;

b) The current $\underline{I_C}$;

c) The current $\underline{I_L}$;

d) The current $\underline{I_T}$;

e) The impedance $\underline{Z_T}$;



a) The current $\underline{I}_R$; $I_R = \frac{E}{R} = \frac{24}{4} = 6A$

b) The current $\underline{I}_C$; $I_c = \frac{E}{X_c} = \frac{24}{6} = 4A$

c) The current $\underline{I}_L$; $I_L = \frac{E}{X_L} = \frac{24}{2} = 12A$

d) The current $\underline{I}_T$; $\underline{I}_T = \underline{I}_R + \underline{I}_L + \underline{I}_C = I_R \times e^{j0} + I_L \times e^{j90} + I_C \times e^{-j90}$

$$\underline{I}_T = I_R + jI_L - jI_C = I_R + j(I_L - I_C)$$

$$I_T = \sqrt{I_R^2 + (I_L - I_c)^2}$$

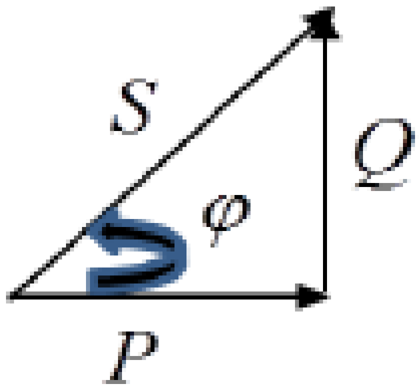
$$I_T = \sqrt{6^2 + 8^2}$$

$$I_T = \sqrt{100} = 10A$$

e) The impedance $\underline{Z}_T$;

$$Z_T = \frac{E}{I_T} = \frac{24}{10} = 2{,}4\Omega$$

Power triangle



$$P = |\underline{S}| \cos \varphi$$

$$Q = |\underline{S}| \sin \varphi$$

$$|\underline{S}| = \sqrt{P^2 + Q^2}$$

Power and Power Factor in an AC Circuit

Active Power

$$P = VI\cos\varphi$$

Reactive Power

$$Q = VI\sin\varphi$$

Apparent Power

$$\left|\underline{S}\right| = VI$$

Power Factor

$$\cos\varphi = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}$$

Compensation with capacities

$$Q_c = -C\omega V^2 = Q' - Q$$

$$-C\omega V^2 = P\tan\varphi' - P\tan\varphi$$

$$C = \frac{P\left(\tan\varphi - \tan\varphi'\right)}{\omega V^2}$$

Complex impedance method for AC circuits (Table 1)

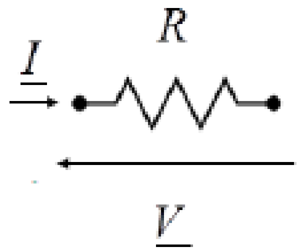
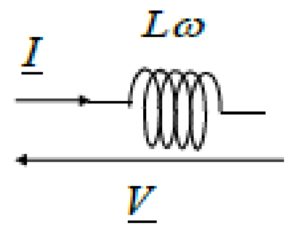
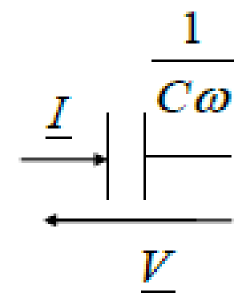
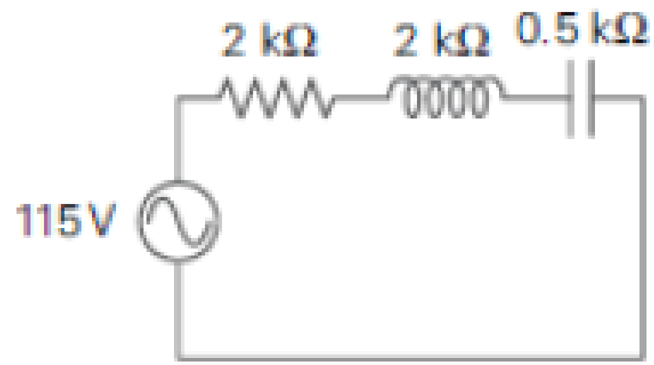
	RESISTOR	INDUCTOR	CAPACITOR
			
Impedance (Ω)	$Z = R$	$Z = L\omega$	$Z = \frac{1}{C.\omega}$
Phase shift (rad)	$\varphi = 0$	$\varphi = +\frac{\pi}{2}$	$\varphi = -\frac{\pi}{2}$
Complex impedance (Ω)	$\underline{Z} = R$	$\underline{Z} = jL\omega$	$\underline{Z} = -j\frac{1}{C.\omega}$
Active Power (W)	$P = R.I^2$	$P = 0$	$P = 0$
Reactive Power (VAR)	$Q = 0$	$Q = L\omega.I^2$	$Q = -C\omega.V^2$
Apparent Power (VA)	$\underline{S} = P$	$\underline{S} = jQ$	$\underline{S} = jQ$

Table 1. Complex impedance

Exercise 2

A circuit is set up as shown below. The frequency of the source is 50Hz
Calculate for the circuit:

a) The current $\underline{I}$;
b) The true power P ;
c) The apparent power S ;
d) The power factor $\cos\varphi$;



Answers:

a) The current $\underline{I}$;

$$I = \frac{V}{Z_T} = \frac{115}{2500} = 0{,}046A$$

$$\underline{Z}_T = R + jX_L - jX_C$$

$$|\underline{Z}_T| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$|\underline{Z}_T| = \sqrt{2^2 + (1{,}5)^2} = 2{,}5k\Omega$$

b) The true power P;

$$P = RI^2 = 2 \times 10^3 \times 0{,}046^2 = 4{,}23W$$

c) The apparent power;

$$S = VI = 115 \times 0{,}046 = 5{,}29VA$$

d) The power factor;

Power Factor (PF): $\cos\varphi = \dfrac{P}{S} = \dfrac{4{,}23}{5{,}29} = 0{,}8$

Review

1. An AC circuit is _______ if inductive reactance and capacitive reactance are equal.
2. A series AC circuit is _______ if there is more inductive reactance than capacitive reactance.
3. A series AC circuit is _______ if there is more capacitive reactance than inductive reactance.
4. For a circuit with a 120 V AC source and a current of 10 A, the apparent power is _________ VA.
6. For a circuit with an apparent power of 3000 VA and a power factor of 0.8, the true power is _______ W.

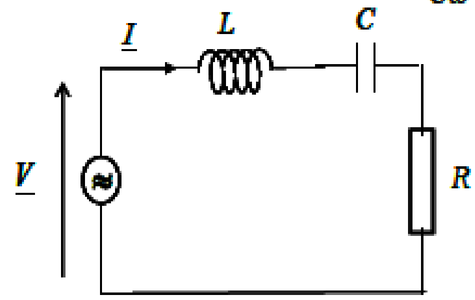
Answers:

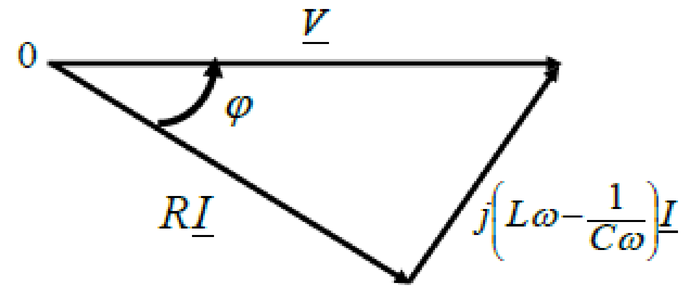
1) Resistive;
2) Inductive;
3) Capacitive;
4) 1200 VA;
6) 2400 W.

Exercise 3

A 100 V AC power supply, 20Ω - resistors, 10Ω coil and -5Ω capacitor are in series.

$\underline{V} = 100e^{j0}V \quad f = 50Hz \quad R = 20\Omega \quad -\frac{1}{C\omega} = -5\Omega \quad L\omega = 10\Omega$





Calculate:

a) The RMS current of $(\underline{I})$ and phase shift (φ)

Answers:

$$\underline{V} = R\underline{I} + jL\omega\underline{I} - j\frac{1}{C\omega}\underline{I} = R\underline{I} + j\underline{I}\left(L\omega - \frac{1}{C\omega}\right)$$

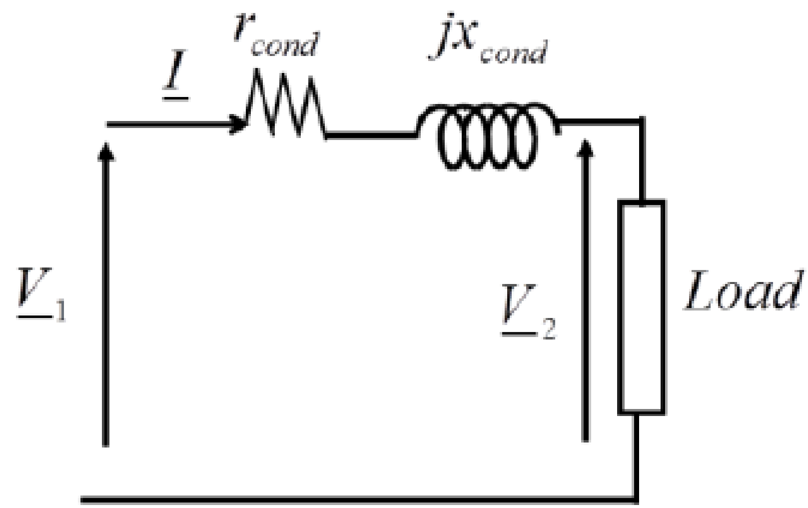
$$X = \left(L\omega - \frac{1}{C\omega}\right); \rightarrow L\omega > -\frac{1}{C\omega} \rightarrow$$

$$X > 0 \rightarrow \underline{Z_{eq}} = 20.615e^{j14} \rightarrow \varphi = 14^\circ = 0{,}244rad$$

$$\underline{V} = \underline{Z_{eq}} \times \underline{I} = 20.615e^{j14} \times \underline{I}$$

$$\underline{I} = \frac{\underline{V}}{\underline{Z_{eq}}} = 4.85e^{-j14} A = 4.85e^{-j0,244} A$$

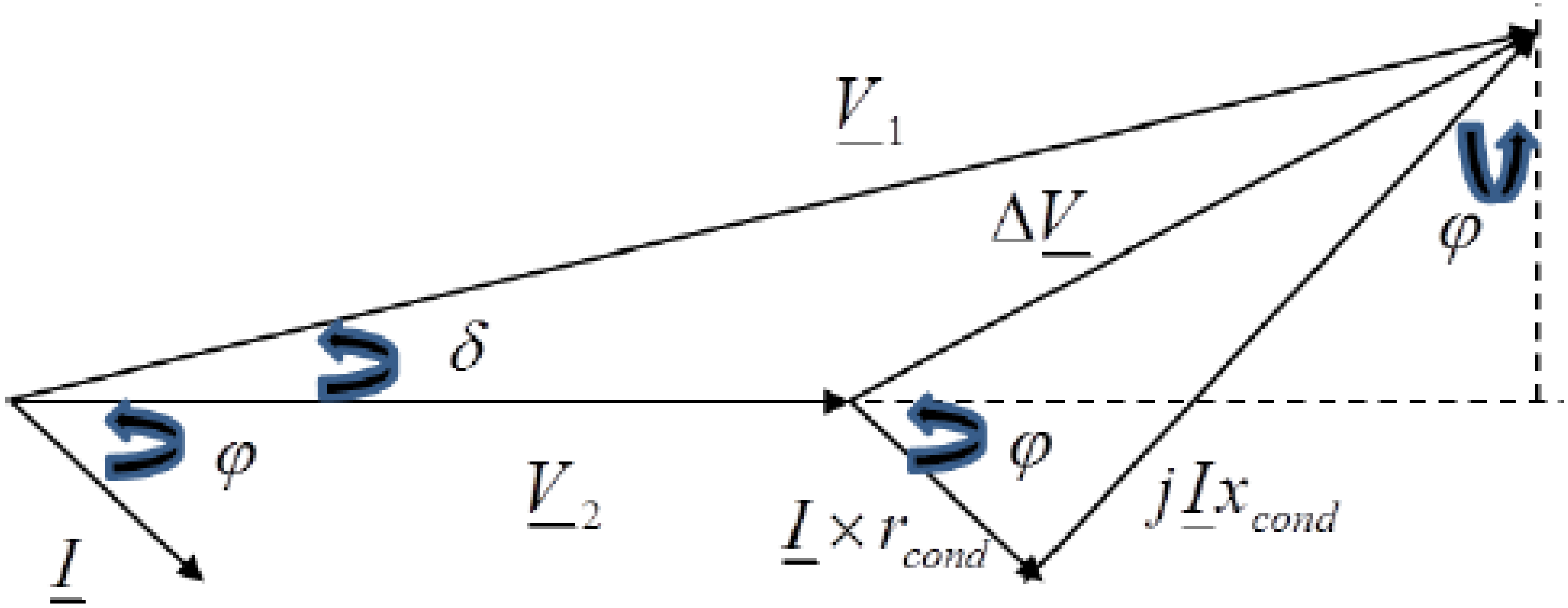
Voltage Drop Calculations



$$\underline{\Delta V} = \underline{Z_{cond}} \times \underline{I}$$

$$\underline{Z_{cond}} = \left(r_{cond} + jx_{cond}\right)$$

$$\underline{\Delta V} = \underline{V_1} - \underline{V_2}$$



$$\begin{cases} V_1\cos\delta = V_2 + r_{cond}I\cos\varphi + x_{cond}I\sin\varphi \\ V_1\sin\delta = -r_{cond}\times I\sin\varphi + x_{cond}\times I\cos\varphi \\ V_1 - V_2 =' r_{cond}I\cos\varphi + x_{cond}I\sin\varphi \rightarrow \delta = 0 \\ \Delta V = r_{cond}I\cos\varphi + x_{cond}I\sin\varphi \end{cases}$$

- **Boucherot theorem or conservation theorem for complex power**

The active power

$$P = \sum_{i=1}^{n} P_i;$$

The reactive power

$$Q = \sum_{i=1}^{n} Q_i$$

The global complex power of the loads becomes then:

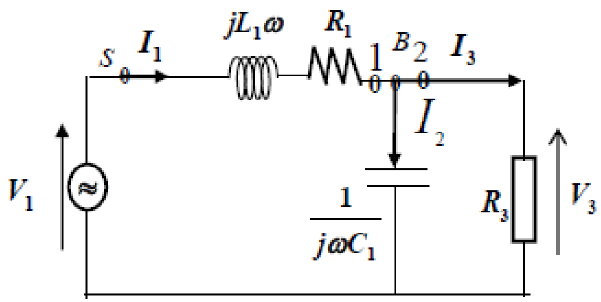
$$\underline{S} = \sum_{i=1}^{n} \underline{S_i}$$

$$\underline{S} = P \pm jQ = \sum_{i=1}^{n} P_i \pm j \sum_{i=1}^{n} Q_i$$

Exercise 4

A circuit is set up as shown below. The frequency of the source is 50Hz. Given $V_3 = 228{,}31V$. Calculate for the circuit:

a) The voltage V_1;



$$V_3 = 228{,}31V; R_1 = 1\Omega; L_1\omega = 2\Omega; \frac{1}{C_1\omega} = 400\Omega; R_3 = 20\Omega$$

Answers

$$P_3 = \frac{V_3^{\;2}}{R_3} = \frac{228{,}31^2}{20} = 2606{,}27W \quad Q_3 = 0 \quad S_3 = \sqrt{P^2{}_3 + Q^2{}_3} = V_3 \times I_3 = 2606{,}27VA$$

$$I_3 = \frac{S_3}{V_3} = 11{,}415A \quad P_1 = 0 + P_3 = 2606{,}27W$$

$$Q_C = -C_1 \omega V_3^{\;2} = -\frac{228{,}31^2}{400} = -130{,}31VAR$$

$$Q_1 = Q_C + Q_3 = -130{,}31VAR \quad S_1 = \sqrt{P^2{}_1 + Q^2{}_1} = V_3 \times I_1 = 2609{,}525VA \quad I_1 = \frac{S_1}{V_3} = 11{,}428A$$

$$P_S = P_1 + R_1 I_1^{\;2} = 2609{,}525 + 1 \times 11{,}424^2 = 2736{,}869W$$

$$Q_S = Q_1 + L_1 \omega I_1^{\;2} = -130{,}31 + 2 * 11{,}428^2 = 130{,}88VAR$$

$$S_S = \sqrt{P_S^{\;2} + Q_S^{\;2}} = V_1 \times I_1 = \sqrt{2736{,}869^2 + 1130{,}88^2} = 2739.996VA$$

$$V_1 = \frac{S_S}{I_1} = 239{,}76V$$

Review

1. _________ is the opposition to current flow in an AC circuit caused by inductance and capacitance.
2. _________ is the total opposition to current flow in an AC circuit with resistance, capacitance, and/or inductance.
3. For a 50 Hz circuit with a 10 mH inductor, the inductive reactance is ________ Ω.
4. In a purely inductive circuit, _________.
a. current and voltage are in phase
b. current leads voltage by 90 degrees
c. current lags voltage by 90 degrees
5. In a purely capacitive circuit, _________.
a. current and voltage are in phase
b. current leads voltage by 90 degrees
c. current lags voltage by 90 degrees
6. For a 50 Hz circuit with a $10\mu F$ capacitor, the capacitive reactance is _________ Ω.
7. A circuit with 5Ω of resistance and 10Ω of inductive reactance has an impedance of _________ Ω.
8. A circuit with 5Ω of resistance and 4Ω of capacitive reactance has an impedance of _________ Ω.

Answers:

1) Reactance;

2) Impedance;

3) 3.14*Ω*;

4) c;

5) b;

6) 318.5*Ω*;

7) 11 .18*Ω*;

8) 6.4*Ω*.


The End