Electrotechnics – HEI 3

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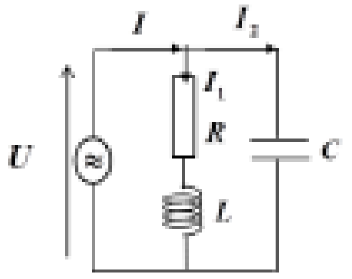
Electrotechnical courses
Part 1

Teaching Methodology

English

Topic

Learning Outcomes



$$\underline{Z_1} = (50 + j62.8) = 80.3e^{j51.5}\Omega$$

Exercise

Review

Keywords

Real component: Partie réelle
Imaginary component: Partie imaginaire
Complex number: Nombres complexes
Complex Plane: Plan complexe
The Cartesian form: Représentation algébrique
The polar form: Représentation exponentielle
The complex conjugate: Nombres complexes conjugués
An imaginary number can't be numerically added to a real number

Basic Formulas

$$\underline{Z} = a + jb = r(\cos\varphi + j\sin\varphi) = r \times e^{j\varphi}$$

$$r = |\underline{Z}| = \sqrt{a^2 + b^2}$$

$$\varphi = arctag(\frac{b}{a})$$

Teaching Methodology

1. **Textbooks of courses**
2. **Textbooks of guided study**
3. **Textbooks of lab experiments**
4. **Intranet**
5. **Internet : Web www.e-lee.eu**

Electrotechnical courses
Part 1

6 theory classes
6 guided study
3 lab experiments

Experiment n°1. Three Phase Transformer
Experiment n°2. Three Phase Induction Machine
Experiment n°3. DC Generator

No student will be allowed to enter in the laboratory unless received the required <u>test</u> for electrical safety

Students will be required to use their own <u>computer</u> for laboratory experiments

Evaluation (English):

A "closed book" examination.
You cannot have in your possession or use: books, notes or paper.

The time allocated for the examination is 2 hours.
Only calculator will be permitted

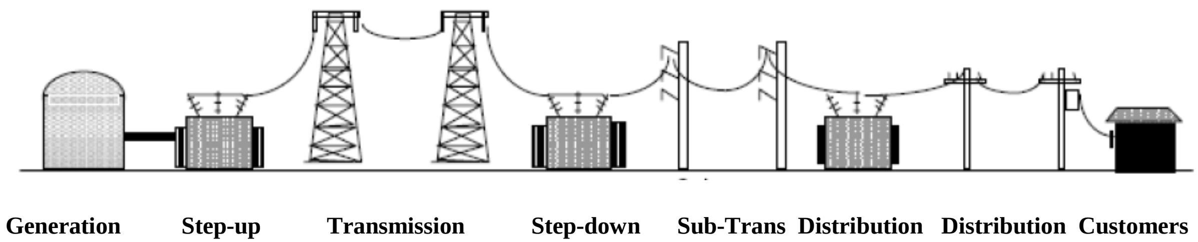
L'épreuve surveillée est (en anglais) , sans document et avec calculatrice et d'une durée de 2 heures.

ELECTROTECHNICS

The study or science of practical and industrial applications of electricity.

Part 1

Electrical Energy Systems



Part 2

DRIVES AND MOTORS

What?

This course is designed to give you an overview of the electrical and electronics technology that is currently used in the industry.

Basic knowledge about electrical circuit theory, circuit analysis methods and operating principles of electric machines

It is not intended to go into sufficient depth because most of you will not even come close to doing such things.

What can I do with Electrotechnical certificate?

It opens up a variety of job perspectives

However, many of you will, <u>as managers</u>, have to deal with the people who do design, run and maintain electronic equipment.

You need <u>to understand</u> what they are talking about so you can discuss problems confidently and understand the technology sufficiently well to specify new equipment and buy appropriate machinery when the time comes.

CHAPTER ONE

Complex Numbers

Learning Outcomes

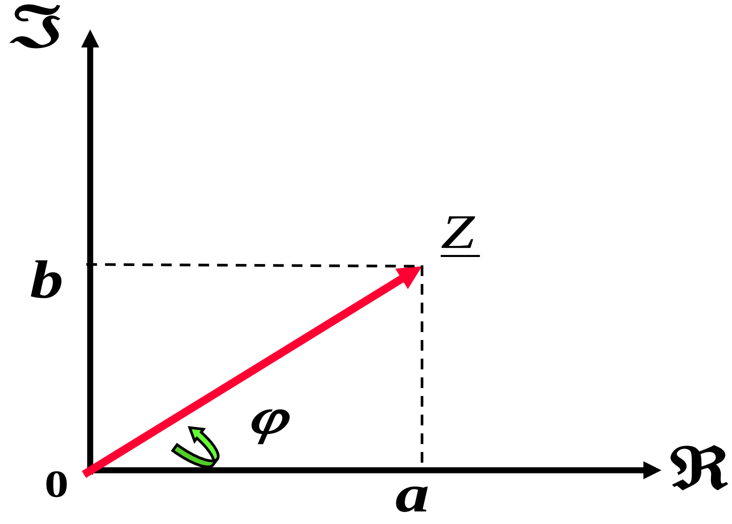
- **add and subtract Complex Numbers**
- **multiply Complex Numbers**
- **divide two Complex Numbers**

Reminder

A complex number $\underline{Z} = a + jb$, consists of the ordered pair (a, b), **a** is the **real** component and **b** is the **imaginary** component.

Note that **a** and **b** are real-valued numbers

The Complex Plane



The Cartesian form

$$\underline{Z} = a + jb$$

The polar form

$$\underline{Z} = r \times e^{j\varphi}$$

Magnitude $$r = \left|\underline{Z}\right| = \sqrt{a^2 + b^2}$$

$$a = \left|\underline{Z}\right| \cos\varphi \qquad b = \left|\underline{Z}\right| \sin\varphi$$

Complex number's angle $$\varphi = arctag\left(\frac{b}{a}\right)$$

Exercise 1.

Convert $\underline{Z} = 3 + 2j$ to polar form.

Answer:

$$\underline{Z} = a + jb$$

$$a = 3; b = 2$$

$$\left|\underline{Z}\right| = r = \sqrt{a^2 + b^2} = \sqrt{13}$$

$$\varphi = \arctan\frac{2}{3} = (33{,}69)^\circ = 0{,}588 rad$$

$$\underline{Z} = \sqrt{13} \times e^{j33{,}69} = \sqrt{13} \times e^{j0{,}588}$$

Addition and subtraction

$$\underline{Z} = \underline{Z_1} + \underline{Z_2} = (a + jb) + (c + jd) = (a + c) + j(b + d)$$

Exercise 2.

What is the sum of a complex number $\underline{Z}$? Given $\underline{Z_1} = 3 - 2j$; $\underline{Z_2} = 5 + 4j$. Required $\underline{Z} = \underline{Z_1} + \underline{Z}_2$

Answer:

$$\underline{Z} = \underline{Z_1} + \underline{Z}_2 = (3 + 5) + (-2 + 4)j = 8 + 2j$$

$$|\underline{Z}| = \sqrt{8^2 + 2^2} = 8{,}24$$

Multiplication

$$\underline{Z} = \underline{Z_1} \times \underline{Z_2}$$

;

$$\underline{Z} = (a + jb) \times (c + jd) = \sqrt{a^2 + b^2} \times \sqrt{c^2 + d^2}\left[(\cos(\varphi_1 + \varphi_2) + j\sin(\varphi_1 + \varphi_2)\right])$$

Exercise 3.

What is the product of a complex number $\underline{Z}$? Given $\underline{Z_1} = \sqrt{3} - j; \underline{Z_2} = 1 - j$. Required $\underline{Z} = \underline{Z_1} \times \underline{Z_2}$

Answer:

$$\underline{Z} = \underline{Z_1} \times \underline{Z_2} \qquad \left|\underline{Z_1}\right| = 2 \qquad \varphi_1 = \frac{-\pi}{6} rad$$

$$\left|\underline{Z_2}\right| = \sqrt{2} \qquad \varphi_2 = \frac{-\pi}{4} rad$$

$$\left|\underline{Z}\right| = \left|\underline{Z_1}\right| \times \left|\underline{Z_2}\right| = 2\sqrt{2} \qquad \varphi = \varphi_1 + \varphi_2 = -\frac{5\pi}{12} rad$$

Division

$$\underline{Z} = \frac{\underline{Z_1}}{\underline{Z_2}} = \frac{a + jb}{c + jd} \qquad |\underline{Z}| = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}} \qquad \varphi = arctg(\frac{b}{a}) - arctg(\frac{d}{c})$$

Exercise 4.

What is the ratio of a complex number $\underline{Z}$? Given $\underline{Z_1} = \sqrt{3} - j; \underline{Z_2} = 1 - j$. Required $\underline{Z} = \frac{\underline{Z_1}}{\underline{Z_2}}$

Answer:

$$|\underline{Z_1}| = 2 \qquad \varphi_1 = \frac{-\pi}{6} rad \qquad |\underline{Z_2}| = \sqrt{2} \qquad \varphi_2 = \frac{-\pi}{4} rad$$

$$|\underline{Z}| = \frac{|\underline{Z_1}|}{|\underline{Z_2}|} = \frac{2}{\sqrt{2}} \qquad \varphi = \varphi_1 - \varphi_2 = \frac{\pi}{12} rad$$

The complex conjugate of $\underline{Z}$, written as $\underline{Z}^*$

$$\underline{Z} = \Re[\underline{Z}] + j\Im[\underline{Z}]$$

$$\underline{Z^*} = \Re[\underline{Z}] - j\Im[\underline{Z}]$$

<u>Example</u>: $\underline{Z} = 3 + 2j$

$$\underline{Z^*} = 3 - 2j$$

Reminder

Most remarkable formula

IF $\varphi = 0$ then $\cos\varphi = 1$, $\sin\varphi = 0$ and $a = |Z|; b = 0$ donc $\underline{Z} = |Z|$

IF $\varphi = +\dfrac{\pi}{2}$ then $\cos\varphi = 0$, $\sin\varphi = 1$ and $b = |Z|; a = 0$ donc $\underline{Z} = j|Z|$

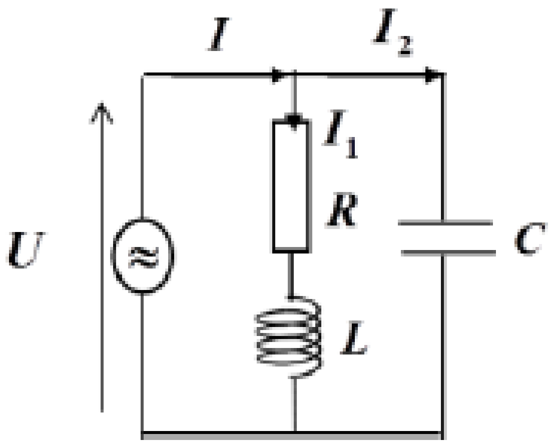
IF $\varphi = -\dfrac{\pi}{2}$ then $\cos\varphi = 0$, $\sin\varphi = -1$ and $b = -|Z|; a = 0$ donc $\underline{Z} = -j|Z|$

Exercise 5.

A circuit is set up as shown below. The frequency of the source is 50Hz

$U = 48V \quad f = 50Hz$

$R = 50\Omega \quad L = 200mH \quad C = 10\mu F$



Calculate for the circuit:

a) The impedance $\underline{Z_1} = (R + jL\omega)$;

b) The impedance $\underline{Z_2} = \dfrac{1}{jC\omega}$

c) The RMS currents and complex number's angle $(\underline{I_1}; \underline{I_2}; \underline{I})$

Answer:

;

a); The impedance $\underline{Z_1} = (R + jL\omega)$;

$$\underline{Z_1} = (50 + j62.8) = 80.3e^{j51.5}\Omega$$

b) The impedance $\underline{Z_2} = \frac{1}{jC\omega}$

$$\underline{Z_2} = \frac{1}{j314 \times 10 \times 10^{-6}} \cong 318e^{-j90}\Omega = 318e^{-j1,57}\Omega$$

c) The RMS currents and complex number's angle $(\underline{I_1}; \underline{I_2}; \underline{I})$

$$\underline{V} = \underline{Z_1} \times \underline{I_1}$$

$$\underline{I_1} = \frac{\underline{V}}{\underline{Z_1}} = \frac{48e^{j0}}{80.3e^{+j51.5}} = 0.598e^{-j51.5}A = 0.598e^{-j0,898}A$$

$$\underline{I}_2 = \frac{\underline{V}}{\underline{Z}_2} = \frac{48e^{j0}}{318e^{-j90}} = 0.151e^{+j90}A = 0.151e^{+j1,57}A$$

$$\underline{I} = \underline{I}_1 + \underline{I}_2 = 0.598e^{-j51.5} + 0.151e^{+j90}$$

$$\underline{I} = (0.372 - j0.468) + j0.151 = 0.372 - j0.317$$

$$\underline{I} = 0.489e^{-j40.4}A = 0.489e^{-j0,705}A$$

$$\underline{Z} = \frac{\underline{V}}{\underline{I}} = \frac{48e^{+j0}}{0.489e^{-j40.4}} = 98.159e^{+j40.4}\Omega = 98.159e^{+j0,705}\Omega$$

$$\varphi = 0,705 rad = 40.4^{\circ} > 0$$

Observation:

$I_1 + I_2 = 0{,}598 + 0{,}151 = 0{,}749A$ It is different from $I = 0{,}489$ A

$\varphi_1 + \varphi_2 = 51{,}5 + (-90) = -38{,}5^{\circ}$ It is different from $\varphi = 40.4^{\circ} > 0$

An imaginary number can’t be numerically added to a real number rather, this notation for a complex number represents vector addition.

Exercise 6

A dipole is designed for use with a $v(t) = 4\sqrt{2}\sin(314t + 0{,}524)$ and $i(t) = 0{,}127\sqrt{2}\sin(314t - 1{,}047)$

Calculate for the circuit:

a) The impedance $\underline{Z}$;

;

b) The inductance L

<u>Answer</u>:

a) The impedance $\underline{Z}$;

$$\varphi = (\theta_u - \theta_i) = 0{,}524 - (-1{,}047) = 1{,}571 rad$$

$$\varphi = 1{,}571 \text{x} \frac{180}{\pi} = +90°$$

$$|Z| = L.\omega = \frac{V}{I} = \frac{4}{0{,}127} = 31{,}5\Omega$$

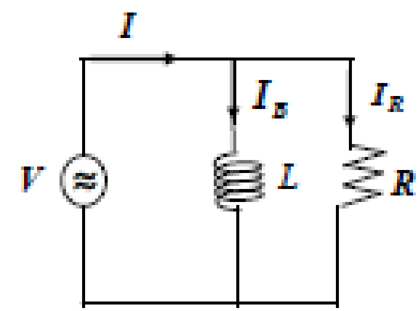
b) The inductance L

$$L = \frac{31{,}5}{\omega} = \frac{31{,}5}{314} = 0{,}1H$$

Exercise 7

;A circuit is set up as shown below. The frequency of the source is 50Hz. Given $I_B = 2A; I_R = 1A$

;



Calculate for the circuit:

a) The RMS current and complex number's angle of $(\underline{I})$

Answer:

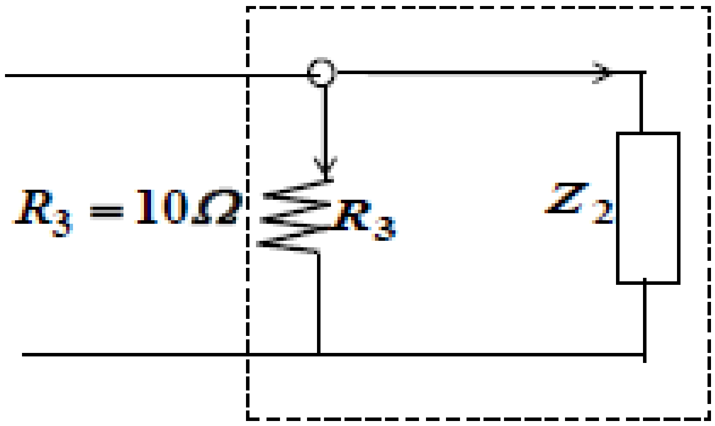
$$I = \sqrt{I_R^2 + I_{\mathbf{B}}^2} = \sqrt{1^2 + 2^2} = \sqrt{5}A \qquad R = \frac{V}{I_R}; L\omega = \frac{V}{I_B}$$

$$\varphi = \tan^{-1}(\frac{Q}{P}) = \tan^{-1}(\frac{L\omega I_B^2}{RI_R^2}) = \tan^{-1}(\frac{I_B}{I_R})$$

$$\varphi = +63.4^{\circ} = +1{,}106rad$$

Exercise 8

A circuit is set up as shown below. The frequency of the source is 50Hz and



$Z_2 = (10 + j15)\Omega$

Calculate for the circuit:

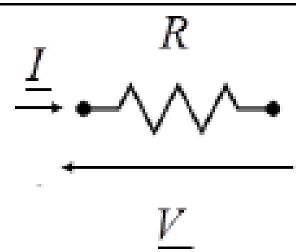
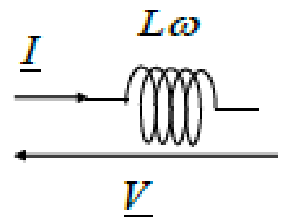
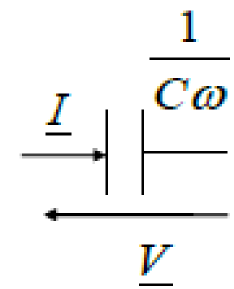
a) The impedance $\underline{Z}$;

<u>Answer</u>:

$$Z = R_3 // Z_2 = \frac{R_3 \times Z_2}{R_3 + Z_2}$$

$$= \frac{10 \times (10 + j15)}{10 + (10 + j15)} = (6.8 + j2.4)\Omega = 7.2e^{j19.4}\Omega = 7.2e^{j0,338}\Omega$$

Complex impedance method for AC circuits (Table 1)

	RESISTOR	INDUCTOR	CAPACITOR
			
Impedance (Ω)	$Z = R$	$Z = L\omega$	$Z = \frac{1}{C.\omega}$
Phase shift (rad)	$\varphi = 0$	$\varphi = +\frac{\pi}{2}$	$\varphi = -\frac{\pi}{2}$
Complex impedance (Ω)	$\underline{Z} = R$	$\underline{Z} = jL\omega$	$\underline{Z} = -j\frac{1}{C.\omega}$
Active Power (W)	$P = R.I^2$	$P = 0$	$P = 0$
Reactive Power (VAR)	$Q = 0$	$Q = L\omega.I^2$	$Q = -C\omega.V^2$
Apparent Power (VA)	$\underline{S} = P$	$\underline{S} = jQ$	$\underline{S} = jQ$


The End