Exercise: Torque control of an induction motor

An asynchronous machine is controlled in IRFO (**Indirect Rotor Field Orientation**). It is a current control; the speed loop is therefore non-existent and the user directly imposes the current references. **The three-phase - diphase transformation used is that of Clarke**.
Dynamic modeling of the three-phase asynchronous machine permits to establish in d-q reference:

$$\begin{cases} v_{ds} = R_s i_{ds} + \sigma L_s \dfrac{di_{ds}}{dt} + \dfrac{M}{L_r}\dfrac{d\varphi_r}{dt} - \omega_s \sigma L_s i_{qs} \\ v_{qs} = R_s i_{qs} + \sigma L_s \dfrac{di_{qs}}{dt} + \omega_s \sigma L_s i_{ds} + \omega_s \dfrac{M}{L_r}\varphi_r \\ 0 = \dfrac{1}{\tau_r}\varphi_r + \dfrac{d\varphi_r}{dt} - \dfrac{M}{\tau_r} i_{ds} \\ 0 = \omega_r \varphi_r - \dfrac{M}{\tau_r} i_{qs} \end{cases}$$

With:
$R_s, L_s, \omega_s, i_{qs}, i_{ds}$ are respectively the resistance, the inductance, the pulsation and the currents of the stator.
φ_r, L_r, ω_r are respectively the flux, the inductance and the pulsation of the rotor.
M : Mutual inductance between stator and rotor.

$$\sigma = 1 - \frac{M^2}{L_s L_r}$$

$\tau_r = \frac{L_r}{R_r}$: Rotor electric time constant
$\tau_s = \frac{L_s}{R_s}$: Stator electric time constant

1- What is this type of control? On which flux vector the axis "d" of the rotating reference should be directed?
2- The term $\frac{M}{L_r}\frac{d\varphi_r}{dt}$ is **neglected** and the terms $\omega_s \sigma L_s i_{qs}$, $\omega_s \sigma L_s i_{ds}$, $\omega_s \frac{M}{L_r}\varphi_r$ are called coupling terms. **They can be compensated.**

Show that the transfer function of the currents of the machine for both axes is:

$$\frac{i_{ds}}{V'_{ds}} = \frac{i_{qs}}{V'_{qs}} = \frac{1}{R_s} \cdot \frac{1}{1 + \sigma \tau_s p} =$$

p is the Laplace operator.
V'_{ds} and V'_{qs} have to be defined.

3- Show that the rotor flux φ_r responds with a first order time constant τ_r. Write its transfer function.
4- Find out how to generate the rotor pulsation ω_r and then the Park angle θ_s.

5- Show that the electromagnetic torque is written :

$$C_e = \frac{3}{2} \mathrm{p} \frac{M}{L_r} \varphi_r i_{qs}$$

From this last expression of the torque, explain the principle of vectorial control with Rotor Field Orientation.

Correction

1) It is an Indirect Rotor Field Orientation: IRFO. We seek to orient the axis d on the rotor flux vector.

2) The term $\frac{M}{L_r}\frac{d\varphi_r}{dt}$ is neglected and the terms $\omega_s \sigma L_s i_{qs}$, $\omega_s \sigma L_s i_{ds}$, $\omega_s \frac{M}{L_r}\varphi_r$ are called coupling terms. They can be compensated by the introduction of calculated terms at the output of the current regulators. These terms are identical with opposite sign. This compensation is only necessary if a high dynamic is required.
Applying the Laplace transformation to system equations, we:

$$v_{ds} + \omega_s \sigma L_s i_{qs} = v'_{ds} = (R_s + \sigma L_s p) i_{ds}$$

$$v_{qs} - \omega_s \sigma L_s i_{ds} - \omega_s \frac{M}{L_r}\varphi_r = v'_{qs} = (R_s + \sigma L_s p) i_{qs}$$

The current transfer function of the machine for both axes is: $\frac{1}{R_s}.\frac{1}{1+\sigma\tau_s p}$.

3) The rotor flux responds with a first-order time constant.

$$0 = \frac{1}{\tau_r}\varphi_r + p\varphi_r - \frac{M}{\tau_r} i_{ds}$$

Thus, the transfer function is: $\varphi_r = \frac{M}{1+p\tau_r} i_{ds}$

4) Generation of the rotor pulsation then the angle of Park:
From the last equation $0 = \omega_r \varphi_r - \frac{M}{\tau_r} i_{qs}$, we calculate rotor pulsation $\omega_r = \frac{i_{qs}}{\tau_r i_{ds}}$
Then angle of Park: $\theta_s = \int \omega_s dt = \int \left(p\Omega + \frac{i^*_{qs}}{\tau_r i^*_{ds}}\right) dt.$
Generally, references rather than measurements are used because of the disturbances (noise, MLI harmonics, etc.) that the measurements may have.

5)

$$\mathrm{p_e(t)} = \omega_s.\frac{M}{L_r}.\varphi_r . i_{qs} = \mathrm{c_e(t)}.\omega_s$$

$$C_e = \frac{3}{2}\mathrm{p}\frac{M}{L_r}\varphi_r i_{qs}$$

Vector control, by decoupling flux and current, simplifies the torque control by making it similar to what happens to a DC machine.