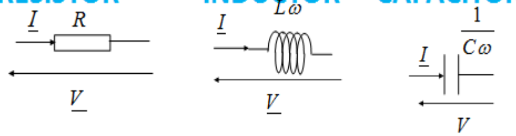
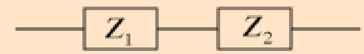
Electrotechnics — Basic notions

Alternative current (AC) Sine Wave , Single-phase AC power



	RESISTOR	INDUCTOR	CAPACITOR
Impedance (Ω)	$\lvert \underline{Z} \rvert = R$	$\lvert \underline{Z} \rvert = L\omega$	$\lvert \underline{Z} \rvert = \frac{1}{C\omega}$
Phase shift (rad)	$\varphi = 0$	$\varphi = +\frac{\pi}{2}$	$\varphi = -\frac{\pi}{2}$
Complex impedance (Ω)	$\underline{Z} = R$	$\underline{Z} = jL\omega$	$\underline{Z} = -j\frac{1}{C\omega}$
Active Power (W)	$P = RI^2$	$P = 0$	$P = 0$
Reactive Power (VAR)	$Q = 0$	$Q = L\omega I^2$	$Q = -C\omega V^2$
Apparent Power (VA)	$\underline{S} = P$	$\underline{S} = jQ$	$\underline{S} = -jQ$

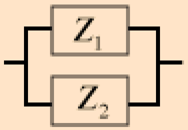
Impedance Combinations



$$Z_1 + Z_2 = (R_1 + jX_1) + (R_2 + jX_2)$$
$$= (R_1 + R_2) + j(X_1 + X_2) = R_{eq} + jX_{eq}$$

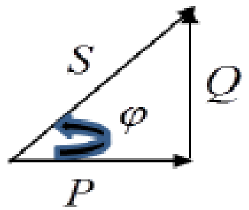
$$\lvert Z \rvert = \sqrt{R_{eq}^2 + X_{eq}^2} \qquad \phi = \tan^{-1}\frac{X_{eq}}{R_{eq}}$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$



$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2} \qquad Z_{eq} = R_{eq} + jX_{eq} = \lvert Z \rvert e^{j\phi}$$

Power triangle



$$P = \lvert \underline{S} \rvert \cos\varphi$$

$$Q = \lvert \underline{S} \rvert \sin\varphi$$

$$\lvert \underline{S} \rvert = \sqrt{P^2 + Q^2}$$

Power and Power Factor in an AC Circuit

$$P = VI\cos\varphi$$

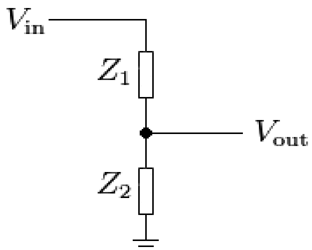
$$Q = VI\sin\varphi$$

$$\lvert \underline{S} \rvert = VI$$

$$\cos\varphi = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}$$

Simple voltage divider

$$V_{out} = \frac{Z_2}{Z_1 + Z_2} \cdot V_{in}$$



Alternative current (AC) Sine Wave , Single-phase AC power

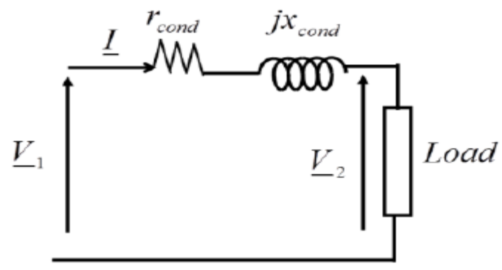
Compensation with capacities

$$Q_c = -C\omega V^2 = Q' - Q$$

$$-C\omega V^2 = P\tan\varphi' - P\tan\varphi$$

$$C = \frac{P(\tan\varphi - \tan\varphi')}{\omega V^2}$$

Voltage drop calculations



$$\underline{\Delta V} = \underline{V_1} - \underline{V_2}$$

$$\underline{\Delta V} = \underline{Z_{cond}} \times \underline{I}$$

$$\underline{Z_{cond}} = (r_{cond} + jx_{cond})$$

$$\Delta V = r_{cond} I\cos\varphi + x_{cond} I\sin\varphi$$

Boucherot theorem or conservation theorem for complex power

$$P = \sum_{i=1}^{n} P_i$$

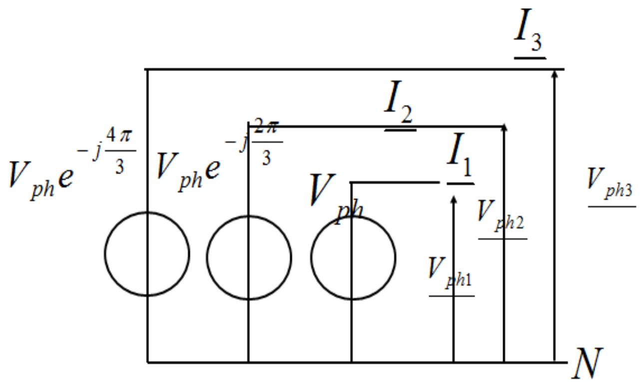
$$Q = \sum_{i=1}^{n} Q_i$$

$$\underline{S} = \sum_{i=1}^{n} \underline{S_i}$$

$$\underline{S} = P \pm jQ = \sum_{i=1}^{n} P_i \pm j \sum_{i=1}^{n} Q_i$$

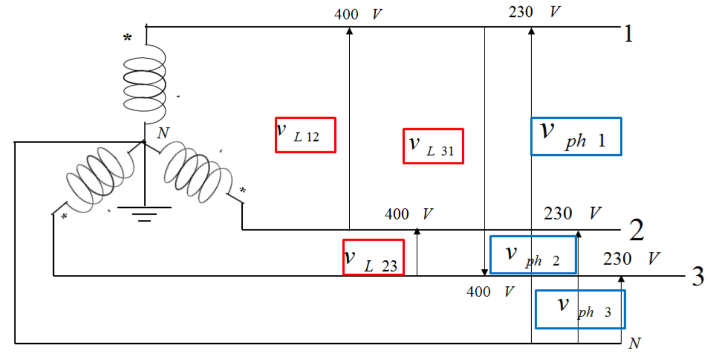
Three-Phase AC POWER

Three phase systems



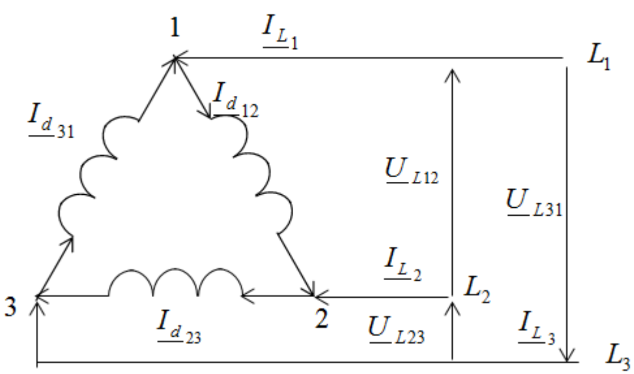
$$\underline{I_1} + \underline{I_2} + \underline{I_3} = 0 \qquad \underline{V_{ph1}} + \underline{V_{ph2}} + \underline{V_{ph3}} = 0$$

Y configuration(Star configuration)



$$V_L = U = V_{ph}\sqrt{3}$$

Delta configuration D

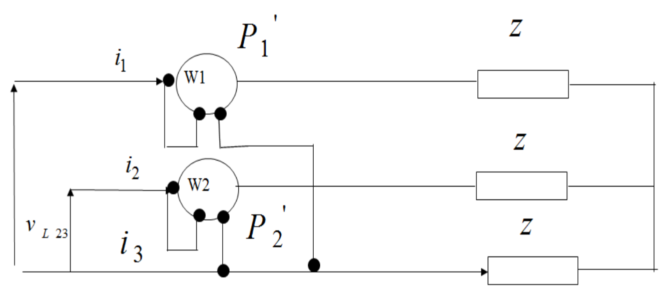


$$I_L = I_d\sqrt{3}$$

Powers : for the two configurations

$$P = \sqrt{3}UI_L\cos\varphi \qquad Q = \sqrt{3}UI_L\sin\varphi \qquad S = \sqrt{3}UI_L$$

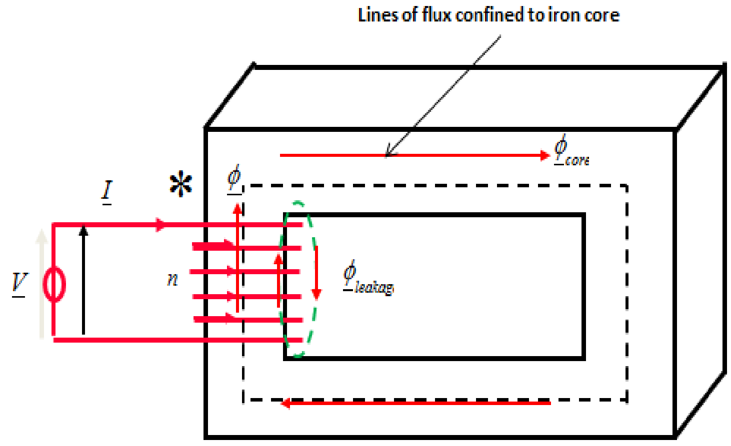
Two-wattmeter configuration



$$P = P_1^{'} + P_2^{'}$$

$$Q = \sqrt{3}\left(P_1^{'} - P_2^{'}\right)$$

Iron Core Inductor

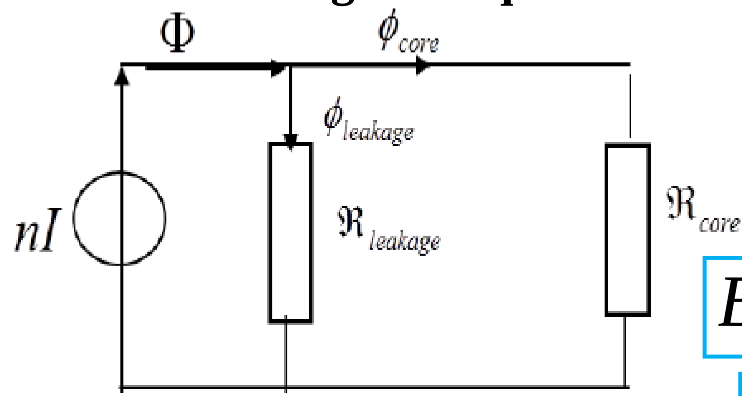


$$\Phi = \phi_{core} + \phi_{leakage}$$

$$\Phi = n \times \phi$$

$$\Phi = B \times S$$

The Magnetic equivalent circuit



$$B = \mu H$$

$$n \times I = \Re \times \phi_{core}$$

$$n \times I = H \times l_{core}$$

$$\mu_r = \frac{\mu}{\mu_0}$$

Self inductance

$$n \times I = \Re \times \phi$$

$$n \times \phi = L \times I \rightarrow \phi = \frac{L \times I}{n}$$

$$n \times I = \Re \times \frac{L \times I}{n}$$

$$L = \frac{n^2}{\Re}$$

Inductance is measured in Henry (H).

Inductive reactance

$$X_L = L\omega = 2\pi f L$$

X_L is measured in (Ohm).

Reluctance

$$\Re = \frac{l_{core}}{\mu \times S}$$

Reluctance is measured in (At/Wb).

Iron Core Inductor

Analogy between magnetic circuit and electric circuit

Magnetic Circuits

- Use Ohm's law analogy to model magnetic circuits

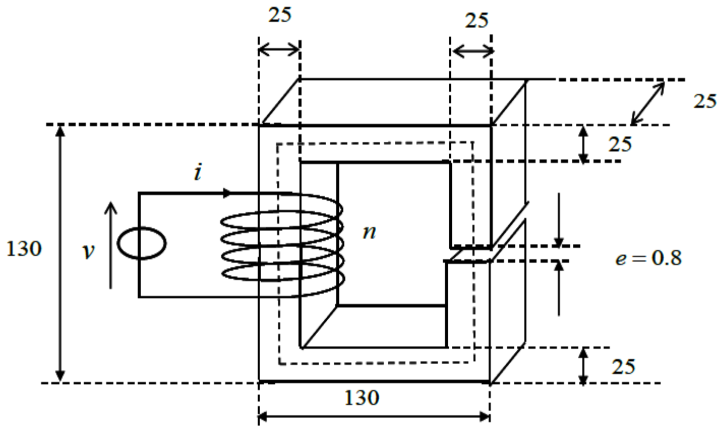
$$V \Leftrightarrow NI$$
$$I \Leftrightarrow \Phi$$
$$R \Leftrightarrow \Re$$

- Use magnetic "reluctance" instead of resistance

$$R = \frac{l}{\sigma A} \Leftrightarrow \Re = \frac{l}{\mu A}$$

Electric quantities	Magnetic quantities
Current I	Magnetic flux Φ
Current density J	Magnetic flux density B
Conductivity σ	Permeability μ
Electromotive force = resistance ×I	Magnetomotive force = reluctance ×Φ
Electric field intensity E	Magnetic field intensity H
Conductance = 1/resistance	Permeance = 1/reluctance
Resistance = l/σ×S	Reluctance = l/μS

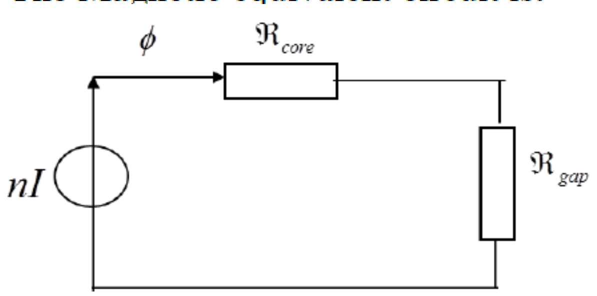
Magnetic system with an Air Gap



$$\Re_{core} = \frac{l_{core}}{\mu_r \times \mu_0 \times S}$$

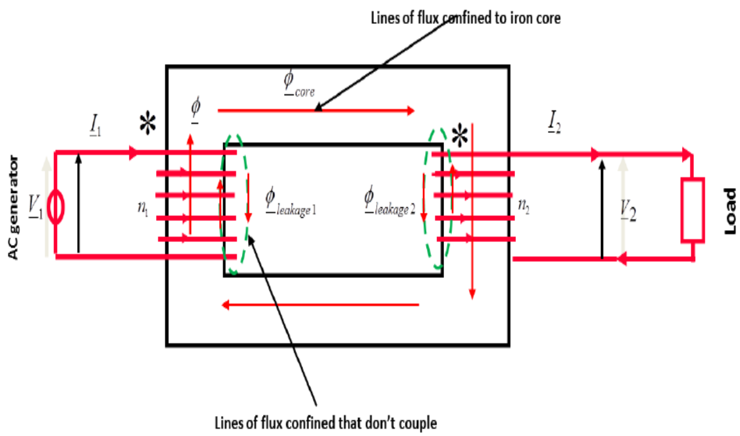
$$\Re_{gap} = \frac{l_{gap}}{\mu_0 \times S}$$

The Magnetic equivalent circuit is:

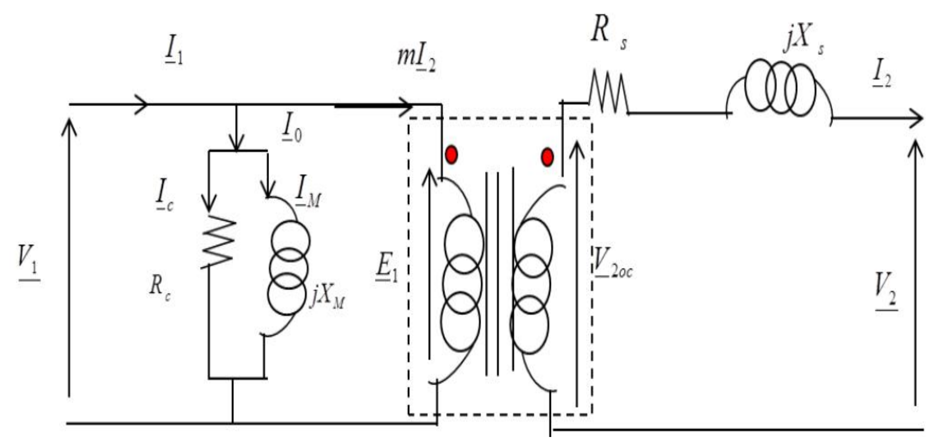


$$nI = \phi\left(\Re_{core} + \Re_{gap}\right)$$

A single phase transformer

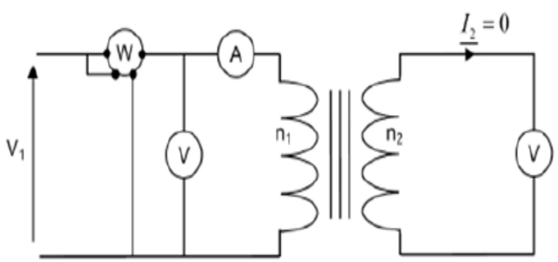


Approximate transformer equivalent circuit



1. Open circuit test

- Performed at **rated voltage**
- Determines **shunt components**



$$m = \frac{n_2}{n_2} = \frac{V_{2oc}}{V_1}$$

$$R_C = \frac{V_1^2}{P_{oc}}$$

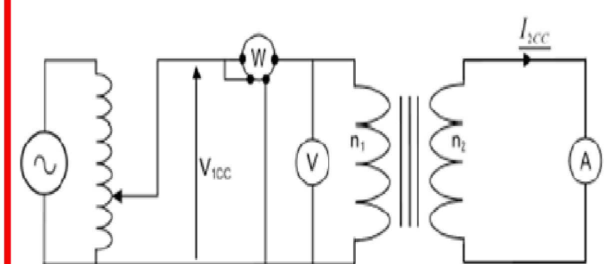
$$X_M = \frac{V_1^2}{Q_M}$$

$$S_{1oc} = V_1 I_{0c}$$

$$Q_M = \sqrt{S_{1oc}^2 - P_{OC}^2}$$

2. Short circuit test

- Performed at **rated current**
- Determines **series components**



$$Z_s = \frac{V_{2sc}}{I_{2sc}}$$

$$R_s = \frac{P_{sc}}{I_{2sc}^2}$$

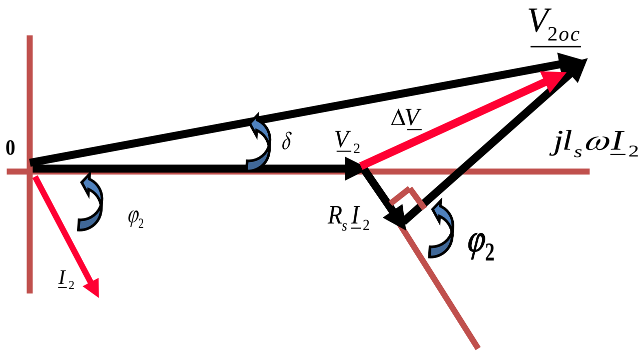
$$X_s = \frac{Q_{sc}}{I_{2sc}^2} = \sqrt{Z_s^2 - R_s^2}$$

$$S_{1sc} = V_{1sc} I_{1sc}$$

$$Q_{sc} = \sqrt{S_{1sc}^2 - P_{sc}^2}$$

A single phase transformer

Voltage drop calculations



$$\Delta V = R_s I_2 \cos\varphi_2 + X_s I_2 \sin\varphi_2$$

Other formulas

$$V_{2oc} = \frac{I_1}{I_2} \times V_1$$

$$m = \frac{n_2}{n_1} = \frac{V_{2oc}}{V_1} = \frac{I_1}{I_2}$$

$$S_r = V_{1r} \times I_{1r} = V_{2oc} \times I_{2r}$$

Condition for Maximum Efficiency

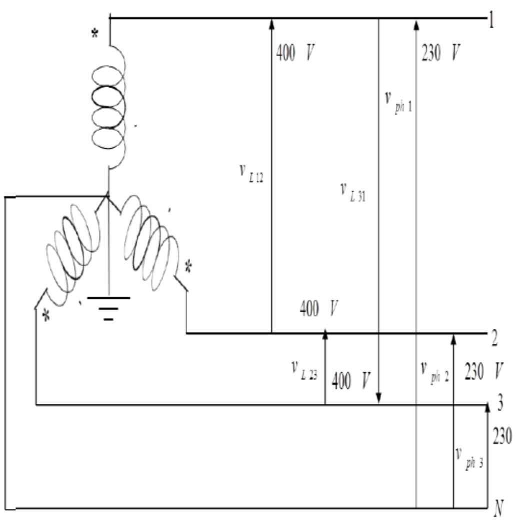
Iron Loss = Copper Loss

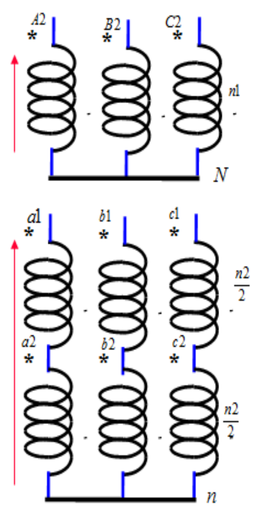
$$p_{core} = p_{cu} = R_s I_2^2$$

$$I_2 = \sqrt{\frac{p_{core} = p_{cu}}{R_s}}$$

Three-Phase Transformers

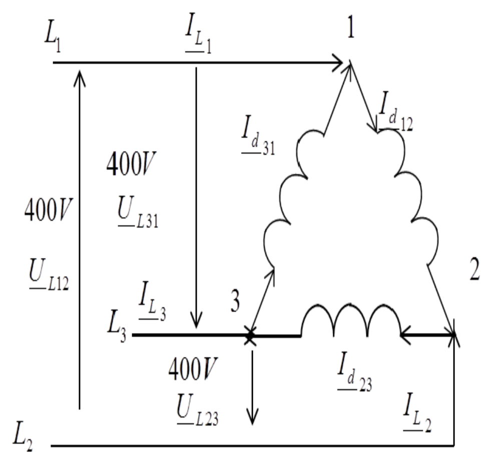
Wye Connections

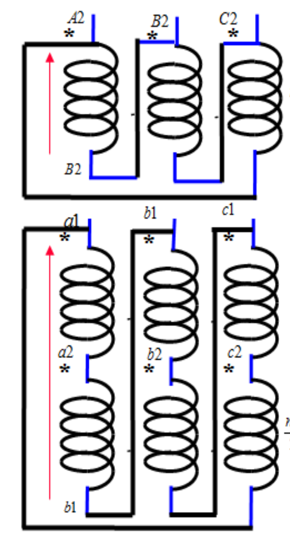




Wye-wye (Yy0)

Delta Connections





Delta-delta (Dd0)

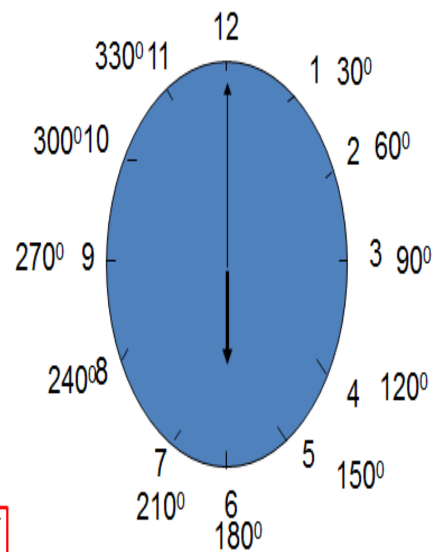
Three-Phase Transformers

Phase displacement

Phase rotation is always counterclockwise (internationally adopted convention) and indicates multiples of 30 degree lag for secondary winding using the primary winding as the reference.

Thus 1 = 30°, 2 = 60°, 3 = 90°, 4 = 120°, 5 = 150°, 6 = 180°, 7 = 210°, 8 = 240°, 9 = 270°, 10 = 300°, 11 = 330° and 12 = 0° or 360°.

$$n = \frac{\theta}{30^\circ}$$

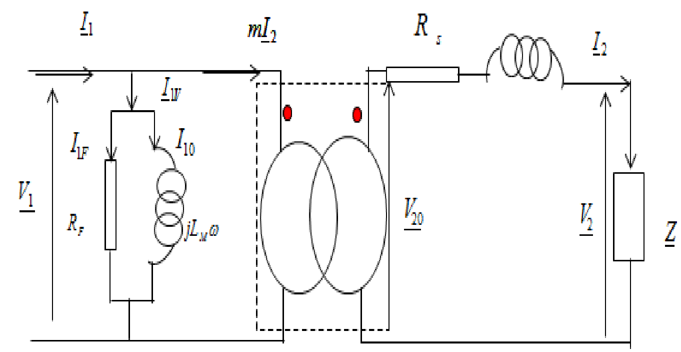


$$n = \frac{180^\circ}{30^\circ} = 6$$

$$\overline{V_a} = m e^{-j\pi} \overline{V_A}$$

Application of Transformer according to Uses: Yy_0, Yd_{11}, Dy_{11}, Dd_0

Single-phase equivalent circuit of 3 phase transformers



$$m = \frac{U_{20}}{U_1} \qquad R_F = \frac{U_1^2}{P_{1V}} \qquad L_M\omega = \frac{U_1^2}{Q_{1V}} \qquad Q_{1V} = \sqrt{S_{1V}^2 - P_{1V}^2} \qquad S_{1V} = \sqrt{3}U_1 I_{1V}$$

$$R_s = \frac{P_{cc}}{3 \times I_{2cc}^2} \qquad l_s\omega = \frac{Q_{cc}}{3 \times I_{2cc}^2} \qquad Q_{cc} = \sqrt{S_{1cc}^2 - P_{cc}^2} \qquad S_{1cc} = \sqrt{3}U_{1cc} I_{1cc}$$

Voltage drop calculations

$$\Delta U_2 = \sqrt{3}\left(R_s I_2 \cos\varphi_2 + l_s \omega I_2 \sin\varphi_2\right)$$

Efficiency

$$\eta = \frac{P_{sortie}}{P_{entrée}} = \frac{\sqrt{3}U_2 I_2 \cos\varphi_2}{\sqrt{3}U_2 I_2 \cos\varphi_2 + P_{1v} + 3 \times R_s \times I_{2cc}^2}$$